# Semantic Theory Semantics Construction (ctd.) 

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## Last Lecture: Semantics

## Construction

- Elementary semantics construction:
- the principle of compositionality
- compositional semantics construction using type theory
- Quantified noun phrases
- Lambda-abstraction and $\beta$-reduction


## The Principle of Compositionality

- The meaning of a complex expression is uniquely determined by the meanings of its sub-expressions and the syntactic rules by which they are combined.
- (The principle is also called "Frege's principle")


## An Example

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# Noun phrases and compositionality 

John works $\Rightarrow$ work'(j*)<br>Somebody works $\Rightarrow \exists x\left(\right.$ work' $\left.^{\prime}(x)\right)$<br>Every student works $\Rightarrow \forall x\left(\right.$ student' $(x) \rightarrow$ work' $\left.^{\prime}(x)\right)$<br>No student works $\Rightarrow \neg \exists x\left(\right.$ student $^{\prime}(x) \wedge$ work' $\left.^{\prime}(x)\right)$<br>John and Mary work $\Rightarrow$ work' $^{\prime}\left(\mathrm{j}^{*}\right)$ ^ work( $\mathrm{m}^{*}$ )

- What's the semantic representation of a noun phrase?


## $\lambda$-Abstraction

- Syntax:
- If $\alpha \in \mathrm{WE}_{\tau}$ and $v \in \mathrm{VAR}_{\sigma}$, then $\lambda v \alpha \in \mathrm{WE}_{(\sigma, \tau)}$.
- Semantics:
- $\llbracket \lambda \vee \alpha \rrbracket^{M, g}$ is that function $f: D_{\sigma} \rightarrow D_{\tau}$ such that for all $a \in D_{\sigma}$, $\mathrm{f}(\mathrm{a})=\llbracket \alpha \rrbracket^{\mathrm{M}, \mathrm{g}[\mathrm{V} / \mathrm{a}]}\left(\right.$ for $\left.\alpha \in \mathrm{WE}_{\tau}, \mathrm{v} \in \mathrm{VAR}_{\sigma}\right)$
- $\left.\mathbb{I}(\lambda \vee \alpha)(\beta) \rrbracket^{M, g}=\llbracket \alpha \rrbracket^{M, g[v / \llbracket} \mathbb{I} \rrbracket M, g\right]$


## Conversion rules in the $\lambda$-calculus

- $\beta$-conversion:
$\lambda v \alpha(\beta) \Leftrightarrow[\beta / v] \alpha$ if all free variables in $\beta$ are free for $v$ in $\alpha$.
- $\alpha$-conversion: $\lambda v \alpha \Leftrightarrow \lambda v^{\prime}\left[v^{\prime} / v\right] \alpha$ if $v^{\prime}$ is free for $v$ in $\alpha$.
- $\eta$-conversion:
$\lambda v(\alpha(v)) \Leftrightarrow \alpha$
- Let $v, v^{\prime}$ be variables of the same type, $\alpha$ any well-formed expression. $v$ is free for $v^{\prime}$ in $\alpha$ iff no free occurrence of $v^{\prime}$ in $\alpha$ is in the scope of a quantifier or a $\lambda$-operator that binds $v$.


## Noun Phrases

$$
\begin{aligned}
\text { John } & \Rightarrow \lambda G\left(G\left(j^{*}\right)\right) \\
\text { Somebody } & \Rightarrow \lambda G \exists x G(x) \\
\text { A student } & \Rightarrow \lambda G \exists x(\operatorname{student}(x) \wedge G(x)) \\
\text { No student } & \Rightarrow \lambda G \neg \exists x(\text { student }(x) \wedge G(x)) \\
\text { John and Mary } & \Rightarrow \lambda G\left(G\left(j^{*}\right) \wedge G\left(m^{*}\right)\right)
\end{aligned}
$$

## "John sleeps"



## Determiners

$$
\begin{aligned}
\text { a, some } & \Rightarrow \lambda F \lambda G \exists x(F(x) \wedge G(x)) \\
\text { every } & \Rightarrow \lambda F \lambda G \forall x(F(x) \neg G(x)) \\
\text { no } & \Rightarrow \lambda F \lambda G \neg \exists x(F(x) \wedge G(x)) \\
\text { most } & \Rightarrow \text { most' }^{\prime} \quad \text { (a constant) }
\end{aligned}
$$

## "Every student works."



## Today

- Semantics construction for further constructions:
- adjectives
- transitive verbs
- Intensional Logic (sketch)


## Back to Adjectives

(1) John is a blond criminal

- criminal'(j*) ^ blond'(j*)
(2) John is a famous criminal
- criminal'(j*) ^ famous'(j*) ?
(3) John is an alleged criminal
- criminal'(j*) ^ alleged'(j*) ???
(4) John is a student


## Back to Adjectives

(1) John is a blond criminal

- blond'(criminal')(j*)
(2) John is a famous criminal
- famous'(criminal')(j*)
(3) John is an alleged criminal
- alleged'(criminal')(j*)
- Now the unwanted inferences disappear ...
(at the price of a less explicit semantic representation)


## "John is a blond criminal."



## Adjective Classes

- Adjectives can be classified with respect to the way they combine the adjective and noun meanings:
- intersective adjectives (blond, carnivorous, ...): $[[$ blond N$]]=[[$ blond ]] $\cap[[\mathrm{N}]]$
- subsective adjectives (skillful, typical, ...): $\left[[\right.$ skillful $N] \subseteq\left[\left[\begin{array}{l}\mathrm{N}]\end{array}\right.\right.$
- privative adjectives (past, fake, ...):
$\left[[\right.$ past $N] \cap\left[\left[\begin{array}{l}N]]=\varnothing \\ \hline\end{array}\right.\right.$
- there are also other non-subsective adjectives that are not privative (alleged, ...)


## Adjectives

- We want:
- compositional semantics construction
- explicit and meaningful semantic representations
- We don't have this yet for (intersective) adjectives.
- We can get this in two different ways
- use meaning postulates
- use more explicit lambda terms


## Meaning Postulates

- Characterise the meaning of a predicate that stands for a word (e.g., "blond") by using logical axioms.
- Meaning postulate for intersective adjectives ("blond"):
- $\forall P \forall x\left(\right.$ blond $\left.{ }^{\prime}(P)(x) \rightarrow P(x)\right)$
- These axioms would be part of our background knowledge.
- For example, we could infer "criminal(john)" from "blond(criminal)(john)" and this axiom.


## More Explicit Lambda Terms

- For intersective adjectives, we can also do it by assigning the word a more elaborate lambda term:
- blond ${ }^{\prime}=\lambda P \lambda x(P(x) \wedge$ blond* $(x))$
- where "blond*" is a constant of type $\langle e, t\rangle$ which should denote the set of blond individuals in the universe.
- This will beta-reduce to the formula we want.


## Transitive Verbs

- A composition problem:

```
every student => \lambdaF \forallx(student'(x) ->F(x)) : \(e,t\rangle,t\rangle
        a paper }=>\lambdaG \existsy(paper'(y)^G(y)): \\langlee,t\rangle,t
        presented = present': <e,(e,t\rangle)
                                    VP
                                    ????
```



```
        present': \e,\langlee,t\rangle\rangle \lambdaG \existsy(paper'(y)^G(y)): \langle(e,t\rangle,t\rangle
        I
        presented
```



## The solution: Type-Raising

- Solution: raise the type of the first-order relation:
- present': $\langle\langle\langle\mathrm{e}, \mathrm{t}\rangle, \mathrm{t}\rangle,\langle\mathrm{e}, \mathrm{t}\rangle\rangle$



## Transitive Verbs

- But now our semantic representation no longer betareduces to a FOL formula.
- $\forall x\left(\right.$ student ${ }^{\prime}(x) \rightarrow \operatorname{present}^{\prime}(\lambda G \exists y$ paper' $\left.(y) \wedge G(y))(x)\right)$
- Same problem as with intersective adjectives, same solution.
- Represent transitive verbs like "present" as follows:
- $\lambda Q \lambda x(Q(\lambda y(p r e s e n t *(y)(x)))):\langle\langle\langle e, t\rangle, t\rangle,\langle e, t\rangle\rangle$,
- where present*: $\langle\mathrm{e},\langle\mathrm{e}, \mathrm{t}\rangle\rangle$


## "... presented a paper"

- a paper $\Rightarrow \lambda G \exists z\left(p^{\prime}{ }^{\prime}(z) \wedge G(z)\right)$
- presented $\Rightarrow \lambda \mathrm{Q} \lambda \times[\mathrm{Q}(\lambda y[$ present* $(\mathrm{y})(\mathrm{x})])]$
- presented a paper
$\left.\Rightarrow \lambda Q \lambda x[Q(\lambda y[p r e s e n t *(y)(x)])]\left(\lambda G \exists z\left(p^{\prime}\right)^{\prime}(z) \wedge G(z)\right)\right)$
$\left.\Rightarrow \lambda x\left[\lambda G \exists z\left(\operatorname{paper}^{\prime}(z) \wedge G(z)\right)\left(\lambda y\left[p^{2}\right)^{*}(y)(x)\right]\right)\right]$
$\Rightarrow \lambda x\left[\exists z\left(p^{\prime} p^{\prime}(z) \wedge \lambda y[p r e s e n t *(y)(x)](z)\right]\right.$
$\Rightarrow \lambda x\left[\exists z\left(\right.\right.$ paper' $^{\prime}(z) \wedge$ present* $\left.^{*}(z)(x)\right]$


## Substitutability

- From the denotational version of the Principle of Compositionality, a substitution principle follows:
- If $A$ is sub-expression in a sentence $S$, and $A$ and $B$ have identical denotations, then A can be replaced by B in S without affecting the truth value of $S$.
(1) George W. Bush is married to Laura Bush.
(2) George W. Bush is the American president
(3) The American president is married to Laura Bush.


## Substitutability?

(1) In 1977, George W. Bush married Laura Bush.
(2) George W. Bush is the American president
(3) In 1977, the American president married Laura Bush.

## Substitutability?

(1) By constitution, the American president is the Supreme Commander of the Armed Forces.
(2) George W. Bush is the American president.
(3) By constitution, George W. Bush is the Supreme Commander of the Armed Forces.

## Substitutability?

(1) Nine necessarily exceeds seven.
(2) Nine is the number of planets
(3) The number of planets necessarily exceeds seven.

## Extensions vs. Intensions

- Two concepts have the same extension if they have the same interpretations:
- "semantics lecture is taking place" and "2 + 2 = 4" are both true right now
- "George W. Bush" and "the US president" refer to the same individual
- However, extensionally equal concepts may still have different "senses:" General truths vs. statements that may become false; can believe in one but not the other...
- These senses are also called intensions.


## Intensions

- We need intensions to explain (non-) substitutability in many contexts:
- propositional attitudes (believe, know, ...)
- indirect speech (say, claim, ...)
- tensed sentences (past, future, ...)
- temporal adverbs (sometimes, always, tomorrow, ...) and connectives (before, during, ...)
- modal adverbs (necessarily, perhaps, ...),
- modal verbs (can, may, must, ...),
- counterfactual conditionals


## Modelling Intensions

- In order to capture the meaning of a NL expression completely, we must extend the logic to talk about intensions.
- Standard technique:
- Introduce the concept of a "possible world";
- define the extension of a term in each possible world;
- the intension is the mapping of possible worlds to extensions.


## Intensional Logics

- Model logic: mechanisms for talking about possible worlds
- $\square \mathrm{p} \quad$ "it is necessarily the case that p" (universal quantification over possible worlds)
- $\diamond p \quad$ "is is possibly the case that $p$ " (existential quantification over possible worlds)


## Intensional Logics

- Temporal logic: mechanisms for talking about time
- Fp "it will at some stage be the case that p"
- Gp "it is always going to be the case that p"
- Pp "it was at some stage the case that p"
- Hp "it always has been the case that p"


## Intensional Logics

- Montagues Intensional Logic (IL)
- model and temporal operators
- plus abstraction over possible worls: ^p denotes the function mapping possible worlds $w$ to the denotation of $p$ at $w$.


## Substitutability, revisited

(1) Nine necessarily exceeds seven.

$$
\square(9>7)
$$

(2) Nine is the number of planets
$9=$ the number of planets
(3) The number of planets necessarily exceeds seven.
$\square$ (the number of planets > 9)

## Substitutability, revisited

(1) John said that Mary kissed Bill.
say"(j*, ^kiss’(m*, b*))
(2) Bill is the smartest boy in class
$x=$ the smartest boy in class
(3) John said that Mary kissed the smartest boy in class. say‘(j*, ^kiss’(m*, the smartest boy in class))

