

## 1 Formalising natural-language sentences

Give formulas of first-order predicate logic that represent the truth conditions of the following sentences as closely as possible. You can ignore tense and use a single predicate symbol to represent several words where appropriate. Definite descriptions (*e.g.*, “the sea”) can be treated like proper names as constants.

1. It is raining.
2. John doesn’t love Mary.
3. Nobody is loved by no one.
4. All students are intelligent.
5. Every intelligent student is successful.
6. If all students are successful, then Mary is successful, too.
7. If Pedro owns a donkey, he beats it.
8. Everybody is annoyed if somebody is noisy.
9. Everybody is annoyed only if Mary is noisy.
10. Dolphins are mammals that live in the sea.

## 2 Status of logical formulas

For each of the following logical sentences, decide whether it is logically valid, unsatisfiable, or contingent. For valid and unsatisfiable sentences, explain why you believe it is valid or unsatisfiable. For contingent sentences, give a model and a countermodel.

1.  $\forall x P(x) \wedge \forall y Q(y) \wedge (\exists z \neg P(z) \vee \exists x \neg Q(x))$
2.  $\forall x P(x) \rightarrow \exists x P(x)$
3.  $\forall x (\text{student}(x) \rightarrow \text{intelligent}(x)) \rightarrow \exists y (\text{student}(y) \wedge \text{intelligent}(y))$
4.  $(\forall x P(x) \wedge \forall y Q(y)) \rightarrow \forall x (P(x) \wedge Q(x))$
5.  $\forall x (\text{student}(x) \rightarrow \text{person}(x))$

## 3 Truth conditions

Calculate the truth conditions of the following formula ( $m^*$  is a constant):

1.  $\forall x (\text{student}(x) \rightarrow \text{intelligent}(x)) \rightarrow \text{intelligent}(m^*)$

## 4 Type Theory: Well-formed expressions

Determine whether the following expressions are well-formed expressions of type theory.  $j$  is a constant of type  $e$ ,  $M$  of type  $\langle e, t \rangle$ ,  $S$  of type  $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$ , and  $C$  of type  $\langle \langle e, t \rangle, t \rangle$ .

1.  $S(M(j))$
2.  $S(M)$
3.  $S(M)(j)$        $[=(S(M))(j)]$
4.  $C(M)$
5.  $C(M)(j)$
6.  $C(S(M))$
7.  $S(S(M))(j)$

## 5 Semantic representations in type theory

Find formulas of type theory that represent the truth conditions of the following sentences *as accurately as possible*, and indicate the types of the constants. You can represent words that are connected by hyphens (“the-book,” “lives-in” etc.) as single constants of an appropriate type. The comments in *italics* are hints that are meant to help you get the intended meaning of the sentences.

1. John gives Mary the-book.
2. Peter owns a red car.
3. The-president rarely sleeps.
4. The-president lives-in a very nice house.
5. Mary eats only a sandwich.  
(*and she doesn't eat anything else*)
6. Mary only EATS a sandwich.  
(*and she doesn't do anything else with it*)

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To be turned in by Wednesday, Mar. 2, 10:00 am at the secretary