Semantic Theory

Lecture 4: Further topics in sentence semantics

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Semantics of λ-expressions

- If $\alpha \in WE_\tau$, $v \in Var_\sigma$, then $[[\lambda v \alpha]]^{M,g}$ is that function $f : D_\sigma \rightarrow D_\tau$ such that for all $a \in D_\sigma$, $f(a) = [[\alpha]]^{M,g[v/a]}$
- Notice that of course $f \in D_{<\sigma,\tau>}$.
- In general: $[[(\lambda v \alpha)(\beta)]]^{M,g} = [[\alpha]]^{M,g[v/ [[\beta]]^{M,g}]}$

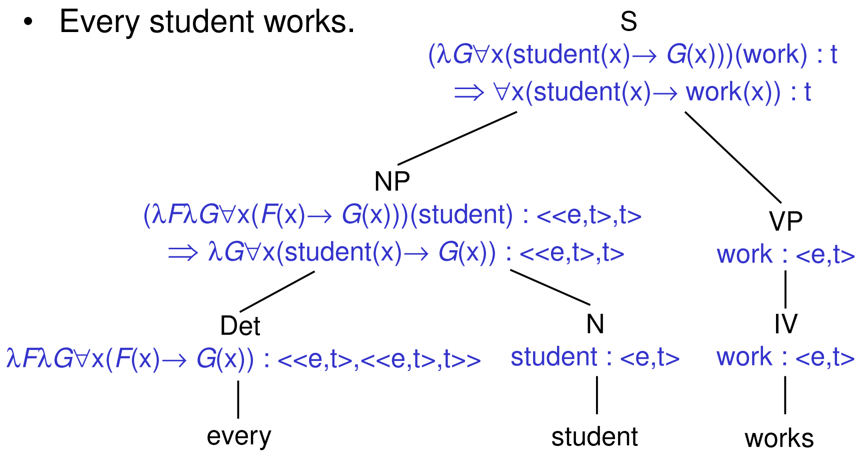
Conversion rules in the λ-calculus

- β-conversion:
  $\lambda v\alpha(\beta) \Leftrightarrow {}^{[\beta/v]}\alpha$, if all free variables in β are free for v in α.
- α-conversion:
  $\lambda v\alpha \Leftrightarrow \lambda v'\,{}^{[v'/v]}\alpha$, if v' is free for v in α.
- η-conversion:
  $\lambda v\alpha(v) \Leftrightarrow \alpha$

The rule which we will use most in semantics construction is β-conversion in the left-to-right direction (β-reduction), which allows us to simplify representations.

Semantics construction

- Every student works.



This week: Conservative extensions

- Semantics construction for further constructions:
  - adjectives
  - transitive verbs
- Extensions to the logic:
  - intensionality
  - tense and modality

The problem with adjective semantics

John is a blond criminal

criminal(j) $\wedge$ blond(j)

John is an honest criminal

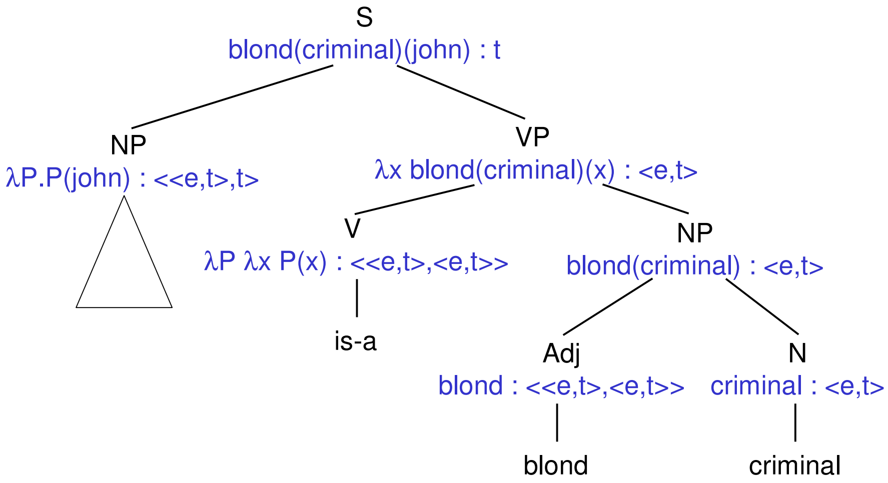
criminal(j) $\wedge$ honest(j) ?

John is an alleged criminal

criminal(j) $\wedge$ alleged(j) ??

... but this one is actually ok!

Another problem with adjective semantics



Adjective classes

- Adjectives can be classified with respect to the way that they combine the adjective and noun meanings:
  - intersective adjectives (blond, carnivorous, ...): $||\text{blond N}|| = ||\text{blond}|| \cap ||\text{N}||$
  - subsective adjectives (skillful, typical, ...): $||\text{skillful N}|| \subseteq ||\text{N}||$
  - privative adjectives (past, fake, ...): $||\text{past N}|| \cap ||\text{N}|| = \varnothing$
  - there are also other non-subsective adjectives that are not privative (alleged, ...)

A new problem with adjectives

- We want the best of both worlds:
  - compositional semantics construction
  - explicit and meaningful final semantic representations
- We don't have this yet for intersective adjectives.
- We can get this in two different ways:
  - use meaning postulates
  - use more explicit lambda terms

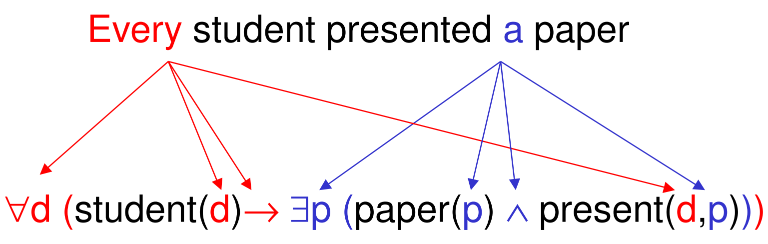
Meaning postulates

- Characterise the meaning of a word by using logical axioms.
- Meaning postulate for intersective adjectives:
  - $\exists P \forall Q \forall x$ blond(Q)(x) $\leftrightarrow$ P(x) $\wedge$ Q(x)
- These axioms would be part of our background knowledge.
- For example, we could infer "criminal(john)" from "blond(criminal)(john)" and this axiom.
- More generally applicable for other words.

More explicit lambda terms

- In the special case of intersective adjectives, we can also do it by assigning the word a more elaborate lambda term:
  - $\lambda P \lambda x\, (P(x) \wedge blond^*(x))$
- This will beta-reduce to the formula we want.
- Note that the symbol "blond*" has type <e,t> here (and should denote the set of blond individuals in the universe), but the entire semantic representation of the word "blond" has type <<e,t>,<e,t>>.

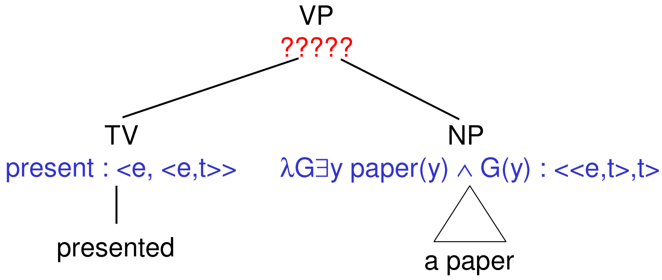
Back to nouns and verbs



Quantificational NPs and transitive verbs

A composition problem:

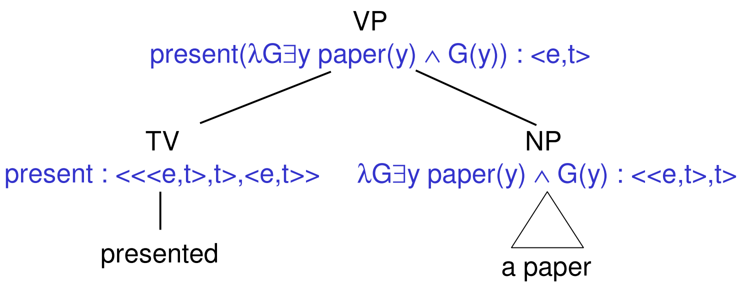
- *every student* ⇒ $\lambda F \forall x(\text{student}(x) \rightarrow F(x))$: <<e,t>,t>
- *a paper* ⇒ $\lambda G\, \exists y(\text{paper}(y) \wedge G(y))$: <<e,t>,t>
- *presented* ⇒ present: <e,<e,t>>



An attempt at a solution

Raise the type of the first-order relation:

present: <<<e,t>,t>,<e,t>>



Spelling out the meaning of a transitive verb

- But now our semantic representation no longer beta-reduces to a FOL formula!

  $\forall x\ student(x) \rightarrow present(\lambda G \exists y\ paper(y) \wedge G(y))(x)$
- Same problem as above, same solution.
- Represent transitive verbs like "present" as follows:

  $\lambda Q\ \lambda x[Q(\lambda y[present^*(y)(x)])]$: <<<e,t>,t>,<e,t>>,

  where present*: <e,<e,t>>
- This is hard to read, but it does the trick.

Spelling out the meaning of a transitive verb

VP

$\lambda Q\ \lambda x[Q(\lambda z[present^*(z)(x)])]\ (\lambda G \exists y\ paper(y) \wedge G(y))$

$\Rightarrow \lambda x\ [\ (\lambda G \exists y\ paper(y) \wedge G(y))\ (\lambda z[present^*(z)(x)])\ ]$

$\Rightarrow \lambda x\ \exists y\ paper(y) \wedge \lambda z[present^*(z)(x)]\ (y)$

$\Rightarrow \lambda x\ \exists y\ paper(y) \wedge present^*(y)(x)$: <e,t>

TV

$\lambda Q\ \lambda x[Q(\lambda z[present^*(z)(x)])]$: <<<e,t>,t>,<e,t>>

presented

NP

$\lambda G \exists y\ paper(y) \wedge G(y)$: <<e,t>,t>

a paper

Non-referential arguments

- John finds a unicorn $|= \exists x\ unicorn'(x)$
- John seeks a unicorn $|\neq \exists x\ unicorn'(x)$

- Subject position of verbs is always referential.
- Direct object position of some verbs is referential, of some other verbs isn't.
- That is, not all transitive verbs can be spelled out in the same way as "present" or "find".
- I.e. there are linguistic reasons why the monster type for transitive verbs makes sense.

Let's look back at compositionality

- Principle of Compositionality:
  - The meaning of a complex expression is uniquely determined by the meanings of its sub-expressions and its syntactic structure.
- But if we assume this, we should be allowed to exchange sub-expressions with the same interpretation without changing the meaning ("salva veritate" substitutability).

Substitutability

- From the denotational version of the Principle of Compositionality, a substitution principle follows:
- If A is sub-expression in sentence C, and A and B have identical denotation, then A can be replaced by B in C without affecting C's truth value.

George W. Bush is married to Laura Bush.

"George W. Bush" = "the American president"

Therefore:

The American president is married to Laura Bush.

Substitutability ?

In 1977, George W. Bush married Laura Bush.

"George W. Bush" = "the American president"

Therefore:

In 1977, the American president married Laura Bush.

??

Substitutability ?

In 1977, the American president was a Democrat.

"George W. Bush" = "the American president"

Therefore:

In 1977, George W. Bush was a Democrat. ???

Substitutability ?

By constitution, the American president is the Supreme Commander of the Armed Forces.

"George W. Bush" = "the American president"

Therefore:

By constitution, George W. Bush is the Supreme Commander of the Armed Forces. ???

Substitutability ?

The following sentences are true:

- The weather is bad
- A semantics lecture is taking place
- 2 + 2 = 4

How about these?

- It is not the case that ...
- Necessarily ...
- Yesterday, it was the case that ...
- John believes that ...

The lesson

- There are expressions that our semantic representations assign the same interpretation, but which cannot always be exchanged for each other without changing the meaning of the sentence.
- Two possibilities:
  - we have to give up compositionality
  - there are meaning distinctions that our representations don't capture

Extensions vs. intensions

- Two concepts have the same extension if they have the same interpretations:
  - "semantics lecture taking place" and "2 + 2 = 4" are both true right now
  - "George W. Bush" and "the US president" refer to the same individual
- However, extensionally equal concepts may still have different "senses": General truths vs. statements that may become false; can believe in one but not the other...
- These senses are also called intensions.

Intensions are everywhere

- If we ignore intensions, we will get into trouble with substitutability in a lot of contexts:
  - propositional attitudes: verbs like "believe", "know", "doubt", "desire", ...
  - verbs of saying: "say", "claim", ...
  - tensed sentences (past, future, ...), temporal adverbs (sometimes, always, lately, tomorrow) and connectives (before, during)
  - modal adverbs (necessarily, perhaps), modal verbs (can, may, must, ...), counterfactual conditionals

Modelling intensions

- In order to capture the meaning of a NL expression completely, we must extend the logic to talk about intensions.
- Standard technique:
  - Introduce the concept of a "possible world";
  - define the extension of a term in each possible world;
  - the intension is the mapping of possible worlds to extensions.

Formal models of intensionality

- Intensional Logic (IL): Extend type theory with
  - mechanisms for talking about possible worlds (modal logic)
  - mechanisms for talking about time (temporal logic)
  - mechanisms for abstracting over possible worlds
- Montague Grammar: use IL in semantics construction (basic ideas as presented here)
- We will now look into modal and temporal aspects in some more detail.

Propositional Modal Logic

- Formulas of propositional modal logic: The smallest set such that:
  - Propositional constants are in For
  - If A, B are in For, so are ¬ A, (A∧B), (A∨B), (A→B),(A↔B), €A, ◊A

Model Structure

- Model structure for propositional modal logic:
  M = <W, V>
  - W is a non-empty set (set of possible worlds)
  - V is value assignment function, which assigns each propositional constant a function W → {0,1}
    For V(p)(w) we also write $V_w(p)$ or $V_{M,w}(p)$.

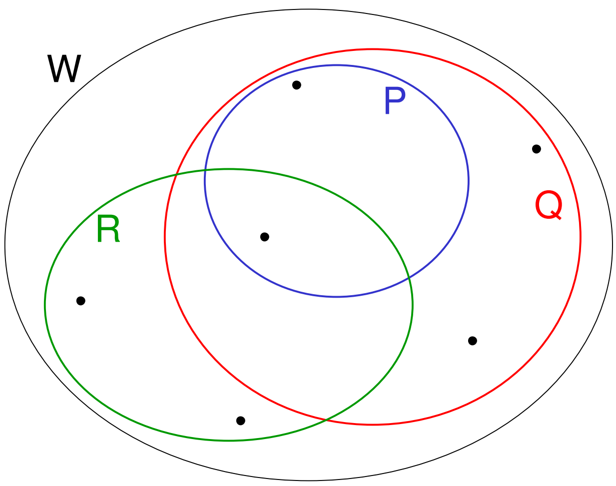
Interpretation

- Interpretation of formulas (with respect to model structure M and possible world w):

$[[p]]^{M,w} = V_M(p)(w)$, if p propositional constant

$[[\neg\varphi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = 0$
$[[\varphi \wedge \psi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = 1$ and $[[\psi]]^{M,w} = 1$
$[[\varphi \vee \psi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = 1$ or $[[\psi]]^{M,w} = 1$
$[[\varphi \rightarrow \psi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = 0$ or $[[\psi]]^{M,w} = 1$
$[[\varphi \leftrightarrow \psi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = [[\psi]]^{M,w}$
$[[\Diamond\, \varphi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w'} = 1$ for at least one $w' \in W$
$[[€\, \varphi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w'} = 1$ for all $w' \in W$

Propositional modal logic



P
◊P
€P
€(P ∨ Q ∨ R)
P → Q

Substitutability ?

Let the following sentences be true:

– The weather is bad

– A semantics lecture is taking place.

– 2+2=4

It is not the case that ...

Necessarily ...

Yesterday, it was the case that ...

John believes that ...

Propositional Tense Logic

- Formulas of propositional tense logic: The smallest set such that:
  – Propositional constants are in For
  – If A, B are in For, so are $\neg$ A, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$,$(A \leftrightarrow B)$, **F**A, **G**A, **P**A, **H**A

FA – "it will at some stage be the case that A"

GA – "it is always going to be the case that A"

PA – "it was at some stage the case that A"

HA – "it always has been the case that A"

Model Structure

- Model structure for propositional tense logic (with linear time):

  M = <T, <, V>

  – T is non-empty set (set of points in time)

  – < is a strict ordering relation on T

  – V is value assignment function, which assigns each propositional constant a function T → {0,1}

  For V(p)(t) we also write $V_t(p)$ or $V_{M,t}(p)$

Interpretation

- Interpretation of formulas (with respect to model structure M and time t):

  $[[p]]^{M,t} = V_M(p)(t)$, if p propositional constant

  $[[\neg\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = 0$

  $[[\varphi \wedge \psi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = 1$ and $[[\psi]]^{M,t} = 1$

  $[[\varphi \vee \psi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = 1$ or $[[\psi]]^{M,t} = 1$

  $[[\varphi \rightarrow \psi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = 0$ or $[[\psi]]^{M,t} = 1$

  $[[\varphi \leftrightarrow \psi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = [[\psi]]^{M,t}$

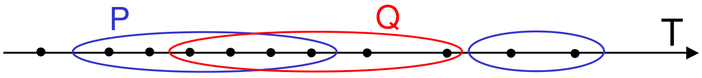
  $[[\mathbf{F}\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t'} = 1$ for at least one t'>t

  $[[\mathbf{G}\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t'} = 1$ for all t'>t

  $[[\mathbf{P}\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t'} = 1$ for at least one t'<t

  $[[\mathbf{H}\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t'} = 1$ for all t'<t

Propositional tense logic



FP
HQ
P(P ∧ **F**Q)
FP ∨ P ∨ **P**P

Substitutability ?

Let the following sentences be true:
– The weather is bad
– A semantics lecture is taking place.
– 2+2=4

It is not the case that ...
Necessarily ...
Yesterday, it was the case that ...
John believes that ...

Propositional Logic with Tense and Modality

- Syntax: Tense + modal operators

- Model structure: M = <W, T, <, V> with
  $V(p): W \times T \rightarrow \{0,1\}$
  alternative notation: $V_{M,w,t}(p)$
- Interpretation with respect to M, w and t.

Semantics for FOL with tense and modalities

- Model structure: M = <U, W, T, <, V>
  - V (or V_M) is value assignment function for non-logical constants, which assigns
    - individuals ($\in U_M$) to individual constants
    - functions $W \times T \rightarrow U^n$ to n-place relational constants
- Assignment function for variables g: $IV \rightarrow U_M$

Interpretation of Terms

- Interpretation of terms (with respect to model structure M and variable assignment g):

  $[[\alpha]]^{M,g,w,t} = V_M(\alpha)$, if α individual constant

  $[[\alpha]]^{M,g,w,t} = g(\alpha)$, if α variable

- Notice: Interpretation of terms doesn't depend on the world and time.

Interpretation

- Interpretation of formulas (with respect to model structure M, variable assignment g, world w and time t):

$[[R(t_1, ..., t_n)]]^{M,g,w,t} = 1$	iff	$\langle [[t_1]]^{M,g,w,t}, ..., [[t_n]]^{M,g,w,t} \rangle \in V_M(R)(w,t)$
$[[s=t]]^{M,g,w,t} = 1$	iff	$[[s]]^{M,g,w,t} = [[t]]^{M,g,w,t}$
$[[\neg\varphi]]^{M,g,w,t} = 1$	iff	$[[\varphi]]^{M,g,w,t} = 0$
$[[\varphi \wedge \psi]]^{M,g,w,t} = 1$	iff	$[[\varphi]]^{M,g,w,t} = 1$ and $[[\psi]]^{M,g,w,t} = 1$
etc.		
$[[\exists x\varphi]]^{M,g,w,t} = 1$	iff	there is $a \in U_M$ such that $[[\varphi]]^{M,g[x/a],w,t} = 1$
$[[\forall x\varphi]]^{M,g,w,t} = 1$	iff	for all $a \in U_M$: $[[\varphi]]^{M,g[x/a],w,t} = 1$
$[[\mathbf{F}\varphi]]^{M,g,w,t} = 1$	iff	$[[\varphi]]^{M,g,w,t'} = 1$ for at least one t'>t
etc.		
[[€A]] $^{M,g,w,t} = 1$	iff	$[[\varphi]]^{M,g,w',t} = 1$ for all $w' \in W$, etc.

Non-Substitutability: Explained!

In 1977, George W. Bush married Laura Bush.

"George W. Bush" = "the American president"

Therefore: I*n 1977, the American president married Laura Bush.* ??

George W. Bush has always been married to Laura Bush.

George W. Bush is the American president.

Therefore: *The American president has always been married to Laura Bush* ???

Non-Substitutability: Explained!

By constitution, the American president is the Supreme Commander of the Armed Forces.

George W. Bush is the American president.

Therefore: *By constitution, George W. Bush is the Supreme Commander of the Armed Forces.* ???

Conclusion

- We extended our type theoretical semantics construction algorithm in two ways:
  - semantics construction for some new semantic phenomena (adjectives, transitive verbs)
  - extension of FOL with intensional constructs (modality, tense)
- Type theory + tense + modality + intensional abstractions = Intensional Logic (not discussed here).