Semantic Theory

Summer 2005

Modal and Tense logic

M. Pinkal / A. Koller

Two levels of interpretation

- Semantic analysis of a NL expression in a logical framework is a two-step process – construction of a semantic representation and its truth-conditional interpretation. Accordingly, the Compositionality Principle has two version:
- The semantic representation of a NL expression is uniquely determined by the semantic representation of its sub-expressions, and the way they are syntactically combined.
- The denotation of a semantic representation is uniquely determined by the denotation of its sub-expressions (and their syntactic combination).

Substitutability

- From the denotational version of the Principle of Compositionality, a substitution principle follows:
- If A is sub-expression in sentence C, and A and B have identical denotation, then A can be replaced by B in C without affecting C's truth value ("salva veritate" substitutability).

George W. Bush is married to Laura Bush.

George W. Bush is the American president.

Therefore: *The American president is married to Laura Bush*

Substitutability ?

George W. Bush has always been married to Laura Bush.

George W. Bush is the American president.

Therefore: *The American president has always been married to Laura Bush ???*

By constitution, the American president is the Supreme Commander of the Armed Forces.

George W. Bush is the American president.

Therefore: *By constitution, George W. Bush is the Supreme Commander of the Armed Forces. ???*

Substitutability ?

Let the following sentences be true:

– The weather is nice

– Bill is working

– 2+2=4

It is not the case that ...

Necessarily ...

Yesterday, it was the case that ...

Extensions and Intensions

- Meanings of expressions are incompletely modelled by interpretation via (extensional) models.
- Moreover, we need the missing parts of the meaning to interpret important types of expressions and constructions:
- Tensed sentences (past, future, ...), temporal adverbs (sometimes, always, lately, tomorrow) and connectives (before, during) require to look into prior and future states of the world.
- Modal adverbs (necessarily, perhaps), modal verbs (can, may, must, ...), counterfactual conditionals require to look into alternative states of the world.
- For a more complete account of meaning information, we define denotations as intensions: functions from points in time / possible worlds to extensions (i.e., truth values, individuals, sets etc.)

Propositional Modal Logic

- Formulas of propositional modal logic: The smallest set such that:
  - Propositional constants are in For
  - If A, B are in For, so are ¬ A, (A∧B), (A∨B), (A→B),(A↔B), €A, ◊A

Model Structure

- Model structure for propositional modal logic:
  M = <W, V>
  - W is a non-empty set (set of possible worlds)
  - V is value assignment function, which assigns each propositional constant a function W → {0,1}
    For V(p)(w) we also write $V_w(p)$ – $V_{M,w}(p)$ respectively

Interpretation

- Interpretation of formulas (with respect to model structure M and possible world w):

$[[p]]^{M,w} = V_M(p)(w)$, if p propositional constant

$[[\neg\varphi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = 0$
$[[\varphi \wedge \psi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = 1$ and $[[\psi]]^{M,w} = 1$
$[[\varphi \vee \psi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = 1$ or $[[\psi]]^{M,w} = 1$
$[[\varphi \rightarrow \psi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = 0$ or $[[\psi]]^{M,w} = 1$
$[[\varphi \leftrightarrow \psi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w} = [[\psi]]^{M,w}$
$[[\Diamond\, \varphi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w'} = 1$ for at least one $w' \in W$
$[[\in \varphi]]^{M,w} = 1$	iff	$[[\varphi]]^{M,w'} = 1$ for all $w' \in W$

Substitutability ?

Let the following sentences be true:

- The weather is nice
- Bill is working
- 2+2=4

It is not the case that ...

Necessarily ...

Yesterday, it was the case that ...

Validity, Entailment, Deduction

- According to the semantics of propositional modal logic, there are additional valid propositions and entailment relations, which again can be cast into deduction calculi.
- Examples: |= €A → A (€A |= A, €A |–A)

  |= A → ◊A

  |= ◊€A → A

  |= €(A ∨ ¬A)

Propositional Tense Logic

- Formulas of propositional tense logic: The smallest set such that:
  - Propositional constants are in For
  - If A, B are in For, so are ¬ A, (A∧B), (A∨B), (A→B),(A↔B), **F**A, **G**A, **P**A, **H**A

FA – "it will at some stage be the case that A"

GA – "it is always going to be the case that A"

PA – "it was at some stage the case that A"

HA – "it always has been the case that A"

Model Structure

- Model structure for propositional tense logic:
  M = <T, <, V>
  - T is non-empty set (set of points in time)
  - < is a strict ordering relation on T
  - V is value assignment function, which assigns each propositional constant a function T → {0,1}
    For V(p)(t) we also write $V_t(p)$ – $V_{M,t}(p)$ respectively

Interpretation

- Interpretation of formulas (with respect to model structure M and time t):
  $[[p]]^{M,t} = V_M(p)(w)$, if p propositional constant
  $[[\neg\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = 0$
  $[[\varphi \wedge \psi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = 1$ and $[[\psi]]^{M,t} = 1$
  $[[\varphi \vee \psi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = 1$ or $[[\psi]]^{M,t} = 1$
  $[[\varphi \rightarrow \psi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = 0$ or $[[\psi]]^{M,t} = 1$
  $[[\varphi \leftrightarrow \psi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t} = [[\psi]]^{M,t}$
  $[[\mathbf{F}\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t'} = 1$ for at least one t'>t
  $[[\mathbf{G}\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t'} = 1$ for all t'>t
  $[[\mathbf{P}\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t'} = 1$ for at least one t'<t
  $[[\mathbf{H}\varphi]]^{M,t} = 1$ iff $[[\varphi]]^{M,t'} = 1$ for all t'<t

Propositional Logic with Tense and Modality

- Syntax: Tense + modal operators

- Model structure: M = <W, T, <, V> with
  - $V(p): W \times T \rightarrow \{0,1\}$
  - alternative notation: $V_{M,w,t}(p)$
- Interpretation with respect to M, w and t.

Semantics for FOL with tense and modalities

- Model structure: M = <U, W, T, <, V>
  - V (V_M) is value assignment function for non-logical constants, which assigns
    - individuals ($\in U_M$) to individual constants
    - functions $W \times T \rightarrow U^n$ to n-place relational constants
- Assignment function for variables $g: IV \rightarrow U_M$

Interpretation of Terms

- Interpretation of terms (with respect to model structure M and variable assignment g):

  $[[\alpha]]^{M,g,w,t} = V_M(\alpha)$, if α individual constant

  $[[\alpha]]^{M,g,w,t} = g(\alpha)$, if α variable

Interpretation

- Interpretation of formulas (with respect to model structure M, variable assignment g, world w and time t):

  $[[R(t_1, ..., t_n)]]^{M,g,w,t} = 1$ iff $\langle [[t_1]]^{M,g,w,t}, ..., [[t_n]]^{M,g,w,t} \rangle \in V_M(R)(w,t)$

  $[[s=t]]^{M,g,w,t} = 1$ iff $[[s]]^{M,g,w,t} = [[t]]^{M,g,w,t}$

  $[[\neg\varphi]]^{M,g,w,t} = 1$ iff $[[\varphi]]^{M,g,w,t} = 0$

  $[[\varphi \wedge \psi]]^{M,g,w,t} = 1$ iff $[[\varphi]]^{M,g,w,t} = 1$ and $[[\psi]]^{M,g,w,t} = 1$

  etc.

  $[[\exists x\varphi]]^{M,g,w,t} = 1$ iff there is $a \in U_M$ such that $[[\varphi]]^{M,g[x/a],w,t} = 1$

  $[[\forall x\varphi]]^{M,g,w,t} = 1$ iff for all $a \in U_M$: $[[\varphi]]^{M,g[x/a],w,t} = 1$

  $[[\mathbf{F}\varphi]]^{M,g,w,t} = 1$ iff $[[\varphi]]^{M,g,w,t'} = 1$ for at least one t'>t

  etc.

  $[[\in A]]^{M,g,w,t} = 1$ iff $[[\varphi]]^{M,g,w',t} = 1$ for all $w' \in W$, etc.

De dicto/de re readings

De dicto/de re readings:

- *The American president has always been married to Laura Bush*

  Only *de re* reading is true.

- *By constitution, the American president is the Supreme Commander of the Armed Forces.*

  Only *de dicto* reading is true.