# Semantic Theory

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## Logic as a framework for NL semantics

- Logic supports precise, consistent and controlled meaning representation via truth-conditional interpretation.
- Logic provides deduction systems to model inference processes, controlled through a formal entailment concept.
- Logic supports uniform modelling of the semantic composition process.

## Syntax of FOL [1]

- Non-logical expressions:
  - Individual constants: IC
  - n-place relation constants: $RC^n$ ($n \geq 0$)
- Individual variables: IV
- Terms: $T = IV \cup IC$
- Atomic formulas:
  - $Rt_1 \ldots t_n$ for $R \in RC^n$, $t_1, \ldots, t_n \in T$
  - $s=t$ for $s, t \in T$



## Syntax of FOL [2]

- FOL formulas: The smallest set For such that:
  - All basic formulas are in For
  - If A, B are in For, so are $\neg A$, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$
  - If x is individual variable, A is in For, so are $\forall xA$, $\exists xA$

## Dolphins in FOL

*Dolphins are mammals, not fish.*

$\forall d\ (dolphin(d) \rightarrow mammal(d)\ \wedge \neg fish(d))$

*Dolphins live in pods.*

$\forall d\ (dolphin(d) \rightarrow \exists x\ (pod(p)\ \wedge\ live\text{-}in\ (d,p))$

*Dolphins give birth to one baby at a time.*

$\forall d\ (dolphin(d) \rightarrow \forall x\ \forall y\ \forall t\ (give\text{-}birth\text{-}to\ (d,x,t)\ \wedge\ give\text{-}birth\text{-}to\ (d,y,t)$
$\rightarrow x=y)$



## Logic as a framework for NL semantics

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- Logic supports uniform modelling of the semantic composition process.

## Semantics of FOL [1]

- Model structure for FOL: M = <U, V>
  - U ($U_M$) is non-empty model universe (individual domain)
  - V ($V_M$) is value assignment function for non-logical constants, which assigns individuals ($\in U_M$) to individual constants and n-tuples of individuals to n-place relation constants
- Assignment function for variables g: IV $\rightarrow$ $U_M$



## Semantics of FOL [2]

- Interpretation of terms (with respect to model structure M and variable assignment g):

  $[[\alpha]]^{M,g} = V_M(\alpha)$, if $\alpha$ individual constant

  $[[\alpha]]^{M,g} = g(\alpha)$, if $\alpha$ variable

# Semantics of FOL [3]

- Interpretation of formulas (with respect to model structure M and variable assignment g):

| | | |
|---|---|---|
| $[[R(t_1, ..., t_n)]]^{M,g} = 1$ | iff | $\langle [[t_1]]^{M,g}, ..., [[t_n]]^{M,g} \rangle \in V_M(R)$ |
| $[[s=t]]^{M,g} = 1$ | iff | $[[s]]^{M,g} = [[t]]^{M,g}$ |
| $[[\neg\varphi]]^{M,g} = 1$ | iff | $[[\varphi]]^{M,g} = 0$ |
| $[[\varphi \wedge \psi]]^{M,g} = 1$ | iff | $[[\varphi]]^{M,g} = 1$ and $[[\psi]]^{M,g} = 1$ |
| $[[\varphi \vee \psi]]^{M,g} = 1$ | iff | $[[\varphi]]^{M,g} = 1$ or $[[\psi]]^{M,g} = 1$ |
| $[[\varphi \rightarrow \psi]]^{M,g} = 1$ | iff | $[[\varphi]]^{M,g} = 0$ or $[[\psi]]^{M,g} = 1$ |
| $[[\varphi \leftrightarrow \psi]]^{M,g} = 1$ | iff | $[[\varphi]]^{M,g} = [[\psi]]^{M,g}$ |
| $[[\exists x\varphi]]^{M,g} = 1$ | iff | there is $a \in U_M$ such that $[[\varphi]]^{M,g[x/a]} = 1$ |
| $[[\forall x\varphi]]^{M,g} = 1$ | iff | for all $a \in U_M$ : $[[\varphi]]^{M,g[x/a]} = 1$ |

- $g[x/a]$ is the variable assignment which is identical with g except that the value for x is substituted by *a*



# Semantics of FOL [4]

- Formula A is true in model structure M

  iff $[[A]]^{M,g} = 1$ for every variable assignment g
- A model structure M satisfies a set of formulas $\Gamma$ (in short: M is a model for $\Gamma$

  iff every formula $A \in \Gamma$ is true in M
- Formula A is valid in FOL

  iff A is true in all FOL model structures

## Dolphins

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## Students

*Every student presented a paper*

$\forall s\ (student(s) \rightarrow \exists p\ (paper(p) \wedge present(s,p)))$

*The students know each other*

*The students help each other*

*The students are sitting aside of each other*

## Logic as a framework for NL semantics

- Logic supports precise, consistent and controlled meaning representation via truth-conditional interpretation.
- Logic provides deduction systems to model inference processes, controlled through a formal entailment concept.
- Logic supports uniform modelling of the semantic composition process.



## Entailment and Deduction [1]

- A set of formulas $\Gamma$ entails formula A:
  $\Gamma \models A$ iff A is true in every model of $\Gamma$
- (Standard) FOL has one semantics, and therefore a unique semantics-based entailment concept.
- Reasoning via computation of truth-conditions is infeasible, or at the very least inconvenient.
- Reasoning/inference is the task of deduction systems.
- There is a wide variety of FOL deduction systems:
  - Axiomatic deduction
  - Gentzen calculus (natural deduction)
  - Tableau deduction
  - Resolution
  - ...

## Entailment and Deduction [2]

- A deduction system is a collection of symbol manipulation rules that derive formulas from sets of formulas (premisses).
- Derivability of A from $\Gamma$ in a deduction system S is written as

  $\Gamma \mid\!-_S A$ (or simply $\Gamma \mid\!- A$)
- Provability of A from $\Gamma$ in S: $\mid\!-_S A$ ($\Gamma \mid\!- A$)
- A simple example for a deduction rule is Modus Ponens:

  $A, A \rightarrow B \mid\!- B$



## Entailment and Deduction [3]

- A FOL deduction system S is sound iff the following holds:

  If $\Gamma \mid\!-_S A$, then $\Gamma \models A$
- A FOL deduction system S is complete iff the following holds:

  If $\Gamma \models A$, then $\Gamma \mid\!-_S A$
- Soundness of a deduction system S guarantees that a false conclusion will never be derived by S from true premisses.
- Completeness guarantees that every sentence entailed by a set of premisses can in principle be derived by S.

## <Notes>

- Deduction system + theorem prover
- Satisfiability, consistency, provability, validity
- Entailment/deduction based on theories/knowledge bases:
  - Ontologies, lexical-semantic databases: WordNet
  - World Knowledge
  - Delimitation of semantic knowledge and world knowledge
- Description logic
- Default reasoning



## Logic as a framework for NL semantics

- Logic supports precise, consistent and controlled meaning representation via truth-conditional interpretation.
- Logic provides deduction systems to model inference processes, controlled through a formal entailment concept.
- Logic supports uniform modelling of the semantic composition process.

## Logic as a framework for NL semantics

- Logic (e.g., FOL) supports precise, consistent and controlled meaning representation via truth-conditional interpretation.
- Logic (e.g., FOL) provides deduction systems to model inference processes, controlled through a formal entailment concept.
- Logic (FOL?) supports uniform modelling of the semantic composition process.



## Frege's Principle

... or the Principle of Compositionality:

- The meaning of a complex expression is uniquely determined by the meanings of its sub-expressions and its syntactic structure.

## Two levels of interpretation

- Semantic interpretation of a NL expression in a logical framework is a two-step process:
  - The NL expression is assigned a semantic representation
  - The semantic representation is truth-conditionally interpreted.
- Truth-conditional interpretation of FOL representations is strictly compositional.
- But what about the computation of a FOL expression for natural-language sentences?



## FOL and Compositionality

*Every student presented a paper*

$$\forall d\ (\mathit{student}(d) \rightarrow \exists p\ (\mathit{paper}(p) \wedge \mathit{present}(d,p)))$$

## FOL and Compositionality

Every student presented a paper

$\forall d\ (student(d) \rightarrow \exists p\ (paper(p) \wedge present(d,p)))$



## The expressive power of FOL [1] – A detour

*John is a married logician*

$logician(j) \wedge married(j)$

## The expressive power of FOL [1] – A detour

*John is a married logician*

logician(j) ∧ married(j)

*John is a graduated logician*

logician(j) ∧ graduated(j) ?



## The expressive power of FOL [1] – A detour

*John is a married logician*

logician(j) ∧ married(j)

*John is a graduated logician*

logician(j) ∧ graduated(j) ?

*John is a passionate logician*

logician(j) ∧ passionate(j) ??

## The expressive power of FOL [2]

*John is driving fast*

drive(j) $\wedge$ fast(j)



## The expressive power of FOL [2]

*John is driving fast*

drive(j) $\wedge$ fast(j)

*John is eating fast*

eat(j) $\wedge$ fast(j) ??

## The expressive power of FOL [3]

*John is driving very fast.*

*It rains.*
*It rained yesterday.*
*It rains occasionally.*

*Bill is blond. Blond is a hair colour.* (|≠ *Bill is a hair colour.*)



## Type theory

- The types of non-logical expressions provided by FOL – terms and n-ary first-order relations – are not sufficient to describe the semantic function of all natural language expressions.
- Type theory provides a much richer inventory of types – higher-order relations and functions of different kinds.

## Types

- For NL meaning representation the (minimal) set of basic types is {e, t} :
  - e (for entity) is the type of individual terms
  - t (for truth value) is the type of formulas
- All pairs $<\sigma, \tau>$ made up of (basic or complex) types $\sigma$, $\tau$ are types.
- $<\sigma, \tau>$ is the type of functions which map arguments of type $\sigma$ to values of type $\tau$.
- In short: The set of types is the smallest set **T** such that $e, t \in$ **T**, and if $\sigma, \tau \in$ **T,** then also $<\sigma,\tau> \in$ **T.**



## Some useful complex types for NL semantics

- One-place predicate constant: <e,t>
- Two-place relation: <<e,<e,t>>
- Sentence adverbial: <t,t>
- Attributive adjective: <<e,t>,<e,t>>
- Degree modifier: <<<e,t>,<e,t>>,<<e,t>,<e,t>>>

## Second-order predicates

- *Bill is blond. Blond is a hair colour.* (|≠ *Bill is a hair colour.*)



## Some useful complex types for NL semantics

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- Degree modifier: <<<e,t>,<e,t>>,<<e,t>,<e,t>>>
- Second-oder predicate: <<e,t>,t>

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- Degree modifier: <<<e,t>,<e,t>>,<<e,t>,<e,t>>>
- Second-oder predicate: <<e,t>,t>
- Preposition <e, <t,t>>



## Type-theoretic syntax [1]

- Vocabulary:
  - Possibly empty, pairwise disjoint sets of non-logical constants: $Con_\tau$ for every type $\tau$

## Higher-order variables

- *Bill has the same hair colour as John.*
- *Santa Claus has all the attributes of a sadist.*



## Type-theoretic syntax [1]

- Vocabulary:
  - Possibly empty, pairwise disjoint sets of non-logical constants: $Con_\tau$ for every type $\tau$
  - Infinite and pairwise disjoint sets of variables: $Var_\tau$ for every type $\tau$
  - The logical operators known from FOL.

## Type-theoretic syntax[ 2]

- The sets of well-formed expressions $WE_\tau$ for every type $\tau$ are given by:
  - $Con_\tau \subseteq WE_\tau$ for every type $\tau$
  - If $\alpha \in WE_{<\sigma, \tau>}$, $\beta \in WE_\sigma$, then $\alpha(\beta) \in WE_\tau$.
  - If A, B are in $WE_t$, then so are $\neg$ A, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$
  - If A is in $WE_t$, then so are $\forall v A$ and $\exists v A$, where *v* is a variable of arbitrary type.
  - If $\alpha$, $\beta$ are well-formed expressions of the same type, then $\alpha = \beta \in WE_t$.



## Examples

*Bill drives fast.*

drive: <e,t>    fast: <<e,t>,<e,t>>

Bill: e    fast(drive): <e,t>

fast(drive)(bill): t

*Mary works in Saarbrücken*

mary: e    work: <e,t>    in: <e,<t,t>>    sb: e

work(mary): t    in(sb): <t,t>

in(sb)(work(mary)): t

## More examples

- *Blond is a hair colour.*
- *Santa Claus has all the attributes of a sadist.*



## Type-theoretic semantics [1]

- Let U be a non-empty set of entities.
- The domain of possible denotations $D_\tau$ for every type $\tau$ is given by:
  - $D_e = U$
  - $D_t = \{0,1\}$
  - $D_{<\sigma, \tau>}$ is the set of functions from $D_\sigma$ to $D_\tau$

## Type-theoretic semantics [2]

- A model structure for a type theoretic language:

  M = <U, V>, where

  – U ($U_M$) is non-empty domain of individuals

  – V ($V_M$) is function, which assigns every non-logical constant of type $\tau$ a member of $D_\tau$ .

- Variable assignment g assigns every variable of type $\tau$ a member of $D_\tau$.



## Type-theoretic semantics [3]

Interpretation (with respect to model structure M and variable assignment g):

$[[\alpha]]^{M,g} = V_M(\alpha)$, if $\alpha$ constant

$[[\alpha]]^{M,g} = g(\alpha)$, if $\alpha$ variable

$[[\alpha(\beta)]]^{M,g} = [[\alpha]]^{M,g}([[\beta]]^{M,g})$

$[[\neg\varphi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 0$

$[[\varphi \wedge \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 1$ and $[[\psi]]^{M,g} = 1$, etc.

If $v \in Var_\tau$, $[[\exists v\varphi]]^{M,g} = 1$ iff there is $a \in D_\tau$ such that $[[\varphi]]^{M,g[v/a]} = 1$

If $v \in Var_\tau$, $[[\forall v\varphi]]^{M,g} = 1$ iff for all $a \in D_\tau$ : $[[\varphi]]^{M,g[v/a]} = 1$

$[[\alpha=\beta]]^{M,g} = 1$ iff $[[\alpha]]^{M,g} = [[\beta]]^{M,g}$

# Semantics of FOL [3]

- Interpretation of formulas (with respect to model structure M and variable assignment g):

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|---|---|---|
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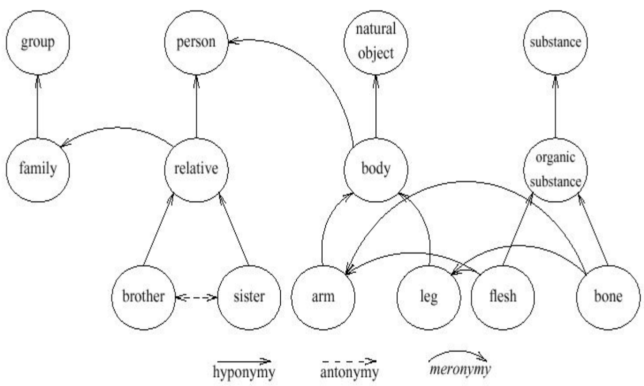
| $[[\varphi \rightarrow \psi]]^{M,g} = 1$ | iff | $[[\varphi]]^{M,g} = 0$ or $[[\psi]]^{M,g} = 1$ |
| $[[\varphi \leftrightarrow \psi]]^{M,g} = 1$ | iff | $[[\varphi]]^{M,g} = [[\psi]]^{M,g}$ |
| $[[\exists x\varphi]]^{M,g} = 1$ | iff | there is h[x]g such that $[[\varphi]]^{M,h} = 1$ |
| $[[\forall x\varphi]]^{M,g} = 1$ | iff | for all h[x]g: $[[\varphi]]^{M,h} = 1$ |

- h[x]g ist short for: h and g are identical except possibly for argument x



# Semantic Relations

Figure 2. Network representation of three semantic relations among an illustrative variety of lexical concepts



## WordNet Senses

The **noun** "body" has 9 senses in WordNet.

1. **body**, organic structure, physical structure -- (the entire physical structure of an organism (especially an animal or human being); "he felt as if his whole body were on fire")
2. **body**, dead body -- (body of a dead animal or person; "they found the body in the lake")
3. **body** -- (a group of persons associated by some common tie or occupation and regarded as an entity; "the whole body filed out of the auditorium")
4. torso, trunk, **body** -- (the body excluding the head and neck and limbs; "they moved their arms and legs and bodies")
5. **body** -- (an individual 3-dimensional object that has mass and that is distinguishable from other objects; "heavenly body")
6. **body** -- (a collection of particulars considered as a system; "a body of law"; "a body of doctrine"; "a body of precedents")
7. **body** -- (the external structure of a vehicle; "the body of the car was badly rusted")
8. consistency, consistence, **body** -- (the property of holding together and retaining its shape; "when the dough has enough consistency it is ready to bake")

9. **body** -- (the central message of a communication; "the body of the message was short")



## WordNet Senses

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## About dolphins

Dolphins are mammals



## About dolphins

Dolphins are mammals, not fish.

## About dolphins

Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf at a time.



## About dolphins

Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf at a time. At birth a bottlenose dolphin calf is about 90-130 cms long and will grow to approx. 4 metres, living up to 40 years.They are highly sociable animals, living in pods which are fairly fluid, with dolphins from other pods interacting with each other from time to time.

## About dolphins



## What should a semantic theory provide?

- A framework to specify word meaning
- The composition process leading from word meanings to sentence information
- The building of a semantic discourse representation from a sequence of sentences in a text (or piece of dialogue)
- Disambiguation/ resolution mechanisms selecting the intended information of an utterance from the large number of linguistically possible interpretations
- Inference mechanisms leading from the given utterance information to other relevant information

## Semantic theory

- How important is semantics?
- What kind of information should a semantic theory model?
- How should semantic information be modelled?
- What is the coverage of logically-based semantic theories?
- How much semantics do we need for language technology?



## Logic as a framework for NL semantics

- Logic provides with its model-theoretic, truth-conditional interpretation is a (so far) indispensable guide to the analysis of natural language meaning.
  - Check whether truth conditions of a logical expression conform with the intuitive judgment about the truth of a sentence in a given situation
- Logic allows to model inference by deduction systems, which again are controlled through a formal entailment concept.
- Logic allows to model the composition process

## Logic as a framework for NL semantics

No good reasons:

- Efficiency
- Cognitive adequacy
- Coverage



## Semantic theory

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## How much of meaning does logic capture?

- Compositional sentence semantics ☺ Type Theory
- Inference ☺ FOL etc.
- Discourse and dialogue semantics ☺ 😐 DRT
- Disambiguation/ resolution ☺☹
- Word meaning ☺☹



## A hard problem for word semantics

## Semantic theory

- How important is semantics?
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- What is the coverage of logically-based semantic theories?
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