Logic as a framework for NL semantics

- Logic supports precise, consistent and controlled meaning representation via truth-conditional interpretation.
- Logic provides deduction systems to model inference processes, controlled through a formal entailment concept.
- Logic supports uniform modelling of the semantic composition process.
Syntax of FOL [1]

• Non-logical expressions:
  – Individual constants: IC
  – n-place relation constants: RC\(^n\) (n ≥ 0)
• Individual variables: IV
• Terms: T = IV∪IC
• Atomic formulas:
  – Rt\(1, \ldots, t_n\) for R\(∈\) RC\(^n\), t\(_1\), ..., t\(_n\) ∈ T
  – s=t for s, t ∈ T

Syntax of FOL [2]

• FOL formulas: The smallest set For such that:
  – All basic formulas are in For
  – If A, B are in For, so are ¬A, (A\(∧\)B), (A\(∨\)B),
    (A→B),(A↔B)
  – If x is individual variable, A is in For, so are ∀xA, ∃xA
Dolphins in FOL

*Dolphins are mammals, not fish.*
\[ \forall d \ (\text{dolphin}(d) \rightarrow \text{mammal}(d) \land \neg \text{fish}(d)) \]

*Dolphins live in pods.*
\[ \forall d \ (\text{dolphin}(d) \rightarrow \exists x \ (\text{pod}(p) \land \text{live-in}(d,p)) \]

*Dolphins give birth to one baby at a time.*
\[ \forall d \ (\text{dolphin}(d) \rightarrow \forall x \ \forall y \ \forall t \ (\text{give-birth-to}(d,x,t) \land \text{give-birth-to}(d,y,t) \rightarrow x=y) \]

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Semantics of FOL [1]

• Model structure for FOL: $M = <U, V>$
  – $U (U_M)$ is non-empty model universe (individual domain)
  – $V (V_M)$ is value assignment function for non-logical constants, which assigns individuals ($\in U_M$) to individual constants and n-tuples of individuals to n-place relation constants
• Assignment function for variables $g: IV \rightarrow U_M$

Semantics of FOL [2]

• Interpretation of terms (with respect to model structure $M$ and variable assignment $g$):
  $[[\alpha]]^M_g = V_M(\alpha)$, if $\alpha$ individual constant
  $[[\alpha]]^M_g = g(\alpha)$, if $\alpha$ variable
Semantics of FOL [3]

- Interpretation of formulas (with respect to model structure $M$ and variable assignment $g$):
  
  $[[R(t_1, ..., t_n)]^M,g] = 1$ iff $\langle [[t_1]_M,g], ..., [[t_n]_M,g] \rangle \in V_M(R)$
  
  $[[s=t]^M,g] = 1$ iff $[[s]_M,g] = [[t]_M,g]$  
  
  $[[\neg \varphi]^M,g] = 1$ iff $[[\varphi]^M,g] = 0$  
  
  $[[\varphi \wedge \psi]^M,g] = 1$ iff $[[\varphi]^M,g] = 1$ and $[[\psi]^M,g] = 1$
  
  $[[\varphi \vee \psi]^M,g] = 1$ iff $[[\varphi]^M,g] = 1$ or $[[\psi]^M,g] = 1$
  
  $[[\varphi \rightarrow \psi]^M,g] = 1$ iff $[[\varphi]^M,g] = 0$ or $[[\psi]^M,g] = 1$
  
  $[[\varphi \leftrightarrow \psi]^M,g] = 1$ iff $[[\varphi]^M,g] = [[\psi]^M,g]$
  
  $[[\exists x \varphi]^M,g] = 1$ iff there is $a \in U_M$ such that $[[\varphi]^{M,g[x/a]}] = 1$
  
  $[[\forall x \varphi]^M,g] = 1$ iff for all $a \in U_M$ : $[[\varphi]^{M,g[x/a]}] = 1$

- $g[x/a]$ is the variable assignment which is identical with $g$ except that the value for $x$ is substituted by $a$

Semantics of FOL [4]

- Formula $A$ is true in model structure $M$ iff $[[A]^M,g] = 1$ for every variable assignment $g$

- A model structure $M$ satisfies a set of formulas $\Gamma$ (in short: $M$ is a model for $\Gamma$) iff every formula $A \in \Gamma$ is true in $M$

- Formula $A$ is valid in FOL iff $A$ is true in all FOL model structures
Dolphins

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Students

Every student presented a paper
\[ \forall s \ ( \text{student}(s) \rightarrow \exists p \ ( \text{paper}(p) \land \text{present}(s,p))) \]

The students know each other
The students help each other
The students are sitting aside of each other
Logic as a framework for NL semantics

• Logic supports precise, consistent and controlled meaning representation via truth-conditional interpretation.
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Entailment and Deduction [1]

• A set of formulas \( \Gamma \) entails formula \( A \):
  \[ \Gamma \models A \]  iff \( A \) is true in every model of \( \Gamma \)

• (Standard) FOL has one semantics, and therefore a unique semantics-based entailment concept.
• Reasoning via computation of truth-conditions is infeasible, or at the very least inconvenient.
• Reasoning/inference is the task of deduction systems.
• There is a wide variety of FOL deduction systems:
  – Axiomatic deduction
  – Gentzen calculus (natural deduction)
  – Tableau deduction
  – Resolution
  – ...

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Entailment and Deduction [2]

• A deduction system is a collection of symbol manipulation rules that derive formulas from sets of formulas (premisses).
• Derivability of $A$ from $\Gamma$ in a deduction system $S$ is written as:
  $$\Gamma \vdash_S A \quad \text{(or simply } \Gamma \vdash A)$$
• Provability of $A$ from $\Gamma$ in $S$: $\vdash_S A$ ($\Gamma \vdash A$)
• A simple example for a deduction rule is Modus Ponens:
  $$A, A \rightarrow B \vdash B$$

Entailment and Deduction [3]

• A FOL deduction system $S$ is sound iff the following holds:
  $$\text{If } \Gamma \vdash_S A, \text{ then } \Gamma \models A$$
• A FOL deduction system $S$ is complete iff the following holds:
  $$\text{If } \Gamma \models A, \text{ then } \Gamma \vdash_S A$$
• Soundness of a deduction system $S$ guarantees that a false conclusion will never be derived by $S$ from true premisses.
• Completeness guarantees that every sentence entailed by a set of premisses can in principle be derived by $S$. 
<Notes>

- Deduction system + theorem prover
- Satisfiability, consistency, provability, validity
- Entailment/deduction based on theories/knowledge bases:
  - Ontologies, lexical-semantic databases: WordNet
  - World Knowledge
  - Delimitation of semantic knowledge and world knowledge
- Description logic
- Default reasoning

Logic as a framework for NL semantics

- Logic supports precise, consistent and controlled meaning representation via truth-conditional interpretation.
- Logic provides deduction systems to model inference processes, controlled through a formal entailment concept.
- Logic supports uniform modelling of the semantic composition process.
Logic as a framework for NL semantics

- Logic (e.g., FOL) supports precise, consistent and controlled meaning representation via truth-conditional interpretation.
- Logic (e.g., FOL) provides deduction systems to model inference processes, controlled through a formal entailment concept.
- Logic (FOL?) supports uniform modelling of the semantic composition process.

Frege's Principle

... or the Principle of Compositionality:

- The meaning of a complex expression is uniquely determined by the meanings of its sub-expressions and its syntactic structure.
Two levels of interpretation

- Semantic interpretation of a NL expression in a logical framework is a two-step process:
  - The NL expression is assigned a semantic representation
  - The semantic representation is truth-conditionally interpreted.
- Truth-conditional interpretation of FOL representations is strictly compositional.
- But what about the computation of a FOL expression for natural-language sentences?

FOL and Compositionality

\[ \forall d \ (\text{student}(d) \rightarrow \exists p \ (\text{paper}(p) \land \text{present}(d,p))) \]

*Every student presented a paper*
FOL and Compositionality

Every student presented a paper

∀d (student(d) → ∃p (paper(p) ∧ present(d, p)))

The expressive power of FOL [1] – A detour

John is a married logician

logician(j) ∧ married(j)
The expressive power of FOL [1] – A detour

John is a married logician
\[ \text{logician}(j) \land \text{married}(j) \]

John is a graduated logician
\[ \text{logician}(j) \land \text{graduated}(j) \]

John is a passionate logician
\[ \text{logician}(j) \land \text{passionate}(j) \]
The expressive power of FOL [2]

*John is driving fast*

\[ \text{drive}(j) \land \text{fast}(j) \]

*John is eating fast*

\[ \text{eat}(j) \land \text{fast}(j) \]
The expressive power of FOL [3]

*John is driving very fast.*

*It rains.*
*It rained yesterday.*
*It rains occasionally.*

*Bill is blond. Blond is a hair colour. (≠ Bill is a hair colour.)*

Type theory

- The types of non-logical expressions provided by FOL – terms and n-ary first-order relations – are not sufficient to describe the semantic function of all natural language expressions.
- Type theory provides a much richer inventory of types – higher-order relations and functions of different kinds.
Types

- For NL meaning representation the (minimal) set of basic types is \{e, t\}:
  - e (for entity) is the type of individual terms
  - t (for truth value) is the type of formulas
- All pairs \(<\sigma, \tau>\) made up of (basic or complex) types \(\sigma, \tau\) are types.
- \(<\sigma, \tau>\) is the type of functions which map arguments of type \(\sigma\) to values of type \(\tau\).
- In short: The set of types is the smallest set \(T\) such that \(e, t \in T\), and if \(\sigma, \tau \in T\), then also \(<\sigma, \tau> \in T\).

Some useful complex types for NL semantics

- One-place predicate constant: \(<e, t>\)
- Two-place relation: \(<<e, e, t>>\>
- Sentence adverbial: \(<t, t>\>
- Attributive adjective: \(<<e, t>, <e, t>>\>
- Degree modifier: \(<<<e, t>, <e, t>>, <<e, t>, <e, t>>>>\>
Second-order predicates

- \textit{Bill is blond. Blond is a hair colour. (\(\neq\) Bill is a hair colour.)}

Some useful complex types for NL semantics

- One-place predicate constant: \(<e,t>\)
- Two-place relation: \(<<e,<e,t>>>\)
- Sentence adverbial: \(<t,t>\)
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- Degree modifier: \(<<<e,t>,<e,t>>,<<e,t>,<e,t>>>\)
- Second-order predicate: \(<<e,t>,t>\)
Some complex types useful for NL semantics

• One-place predicate constant: \(<e,t>\)
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• Attributive adjective: \(<<e,t>,<e,t>>\>
• Degree modifier: \(<<<e,t>,<e,t>>,<<e,t>,<e,t>>>>\>
• Second-order predicate: \(<<e,t>,t>\>
• Preposition \(<e, <t,t>>\>\)
Higher-order variables

• *Bill has the same hair colour as John.*

• *Santa Claus has all the attributes of a sadist.*

Type-theoretic syntax [1]

• Vocabulary:
  – Possibly empty, pairwise disjoint sets of non-logical constants: \( \text{Con}_\tau \) for every type \( \tau \)
  – Infinite and pairwise disjoint sets of variables: \( \text{Var}_\tau \) for every type \( \tau \)
  – The logical operators known from FOL.
Type-theoretic syntax[2]

- The sets of well-formed expressions $\text{WE}_\tau$ for every type $\tau$ are given by:
  - $\text{Con}_\tau \subseteq \text{WE}_\tau$ for every type $\tau$
  - If $\alpha \in \text{WE}_{<\sigma, \tau>}$, $\beta \in \text{WE}_\sigma$, then $\alpha(\beta) \in \text{WE}_\tau$
  - If $A, B$ are in $\text{WE}_I$, then so are $\neg A$, $(A \land B)$, $(A \lor B)$, $(A \rightarrow B)$
  - If $A$ is in $\text{WE}_I$, then so are $\forall \nu A$ and $\exists \nu A$, where $\nu$ is a variable of arbitrary type.
  - If $\alpha, \beta$ are well-formed expressions of the same type, then $\alpha = \beta \in \text{WE}_I$

Examples

Bill drives fast.

- drive: $<e,t>$  fast: $<e,t>,<e,t>$
- Bill: $e$  fast(drive): $<e,t>$
- fast(drive)(bill): $t$

Mary works in Saarbrücken

- mary: $e$  work: $<e,t>$  in: $<e,<t,t>>$  sb: $e$
- work(mary): $t$  in(sb): $<t,t>$
- in(sb)(work(mary)): $t$
More examples

- *Blond is a hair colour.*

- *Santa Claus has all the attributes of a sadist.*

Type-theoretic semantics [1]

- Let $U$ be a non-empty set of entities.
- The domain of possible denotations $D_\tau$ for every type $\tau$ is given by:
  - $D_e = U$
  - $D_i = \{0,1\}$
  - $D_{<\sigma, \tau>}$ is the set of functions from $D_\sigma$ to $D_\tau$
Type-theoretic semantics [2]

• A model structure for a type theoretic language:
  \[ M = \langle U, V \rangle, \text{ where} \]
  \[ - U (U_M) \text{ is non-empty domain of individuals} \]
  \[ - V (V_M) \text{ is function, which assigns every non-logical} \]
  \[ \text{constant of type } \tau \text{ a member of } D_\tau. \]

• Variable assignment \( g \) assigns every variable of type \( \tau \) a member of \( D_\tau \).

Interpretation (with respect to model structure \( M \) and variable assignment \( g \)):

\[ [[\alpha]]^g_M = V_M(\alpha), \text{ if } \alpha \text{ constant} \]
\[ [[\alpha]]^g_M = g(\alpha), \text{ if } \alpha \text{ variable} \]
\[ [[\alpha(\beta)]]^g_M = [[\alpha]]^g_M([[[\beta]]^g_M)] \]
\[ [[\neg \phi]]^g_M = 1 \text{ iff } [[\phi]]^g_M = 0 \]
\[ [[\phi \land \psi]]^g_M = 1 \text{ iff } [[\phi]]^g_M = 1 \text{ and } [[\psi]]^g_M = 1, \text{ etc.} \]
If \( v \in \text{Var}_\tau \), \[ [[\exists v \phi]]^g_M = 1 \text{ iff there is } a \in D_\tau \text{ such that } [[\phi]]^g_M[v/a] = 1 \]
If \( v \in \text{Var}_\tau \), \[ [[\forall v \phi]]^g_M = 1 \text{ iff for all } a \in D_\tau : [[\phi]]^g_M[v/a] = 1 \]
\[ [[\alpha = \beta]]^g_M = 1 \text{ iff } [[\alpha]]^g_M = [[\beta]]^g_M \]
Semantics of FOL [3]

- Interpretation of formulas (with respect to model structure $M$ and variable assignment $g$):
  
  \[ [[R(t_1, \ldots, t_n)]]_{M,g} = 1 \quad \text{iff} \quad \langle [[t_1]]_g, \ldots, [[t_n]]_g \rangle \in V_M(R) \]
  
  \[ [[s=t]]_{M,g} = 1 \quad \text{iff} \quad [[s]]_g = [[t]]_g \]
  
  \[ [[\neg \varphi]]_{M,g} = 1 \quad \text{iff} \quad [[\varphi]]_{M,g} = 0 \]
  
  \[ [[\varphi \land \psi]]_{M,g} = 1 \quad \text{iff} \quad [[\varphi]]_{M,g} = 1 \text{ and } [[\psi]]_{M,g} = 1 \]
  
  \[ [[\varphi \lor \psi]]_{M,g} = 1 \quad \text{iff} \quad [[\varphi]]_{M,g} = 1 \text{ or } [[\psi]]_{M,g} = 1 \]
  
  \[ [[\varphi \rightarrow \psi]]_{M,g} = 1 \quad \text{iff} \quad [[\varphi]]_{M,g} = 0 \text{ or } [[\psi]]_{M,g} = 1 \]
  
  \[ [[\exists x \varphi]]_{M,g} = 1 \quad \text{iff} \quad \text{there is } h[x]_g \text{ such that } [[\varphi]]_{M,h} = 1 \]
  
  \[ [[\forall x \varphi]]_{M,g} = 1 \quad \text{iff} \quad \text{for all } h[x]_g: [[\varphi]]_{M,h} = 1 \]

- $h[x]_g$ is short for: $h$ and $g$ are identical except possibly for argument $x$
WordNet Senses

The noun "body" has 9 senses in WordNet.

1. **body**, organic structure, physical structure -- (the entire physical structure of an organism (especially an animal or human being); "he felt as if his whole body were on fire")
2. **body**, dead body -- (body of a dead animal or person; "they found the body in the lake")
3. **body** -- (a group of persons associated by some common tie or occupation and regarded as an entity; "the whole body filed out of the auditorium")
4. torso, trunk, **body** -- (the body excluding the head and neck and limbs; "they moved their arms and legs and bodies")
5. **body** -- (an individual 3-dimensional object that has mass and that is distinguishable from other objects; "heavenly body")
6. **body** -- (a collection of particulars considered as a system; "a body of law"; "a body of doctrine"; "a body of precedents")
7. **body** -- (the external structure of a vehicle; "the body of the car was badly rusted")
8. consistency, consistence, **body** -- (the property of holding together and retaining its shape; "when the dough has enough consistency it is ready to bake")
9. **body** -- (the central message of a communication; "the body of the message was short")
About dolphins

Dolphins are mammals

Dolphins are mammals, not fish.
About dolphins

Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf at a time. At birth a bottlenose dolphin calf is about 90-130 cms long and will grow to approx. 4 metres, living up to 40 years. They are highly sociable animals, living in pods which are fairly fluid, with dolphins from other pods interacting with each other from time to time.
What should a semantic theory provide?

- A framework to specify word meaning
- The composition process leading from word meanings to sentence information
- The building of a semantic discourse representation from a sequence of sentences in a text (or piece of dialogue)
- Disambiguation/ resolution mechanisms selecting the intended information of an utterance from the large number of linguistically possible interpretations
- Inference mechanisms leading from the given utterance information to other relevant information
Semantic theory

• How important is semantics?
• What kind of information should a semantic theory model?
• How should semantic information be modelled?
• What is the coverage of logically-based semantic theories?
• How much semantics do we need for language technology?

Logic as a framework for NL semantics

• Logic provides with its model-theoretic, truth-conditional interpretation is a (so far) indispensable guide to the analysis of natural language meaning.
  – Check whether truth conditions of a logical expression conform with the intuitive judgment about the truth of a sentence in a given situation
• Logic allows to model inference by deduction systems, which again are controlled through a formal entailment concept.
• Logic allows to model the composition process
Logic as a framework for NL semantics

No good reasons:
• Efficiency
• Cognitive adequacy
• Coverage

Semantic theory

• How important is semantics?
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• How should semantic information be modelled?
• What is the coverage of logically-based semantic theories?
• How much semantics do we need for language technology?
How much of meaning does logic capture?

• Compositional sentence semantics 😊 Type Theory
• Inference 😊 FOL etc.
• Discourse and dialogue semantics 😊 😊 DRT
• Disambiguation/ resolution 😊😊
• Word meaning 😊😊

A hard problem for word semantics

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