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# **Vorlesung Grammatikformalismen**

## **Teil I: Unifikationsgrammatik**

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**Relationen:** unmittelbare Dominanz - Dominanz  
unmittelbare Präzedenz - Präzedenz

**Konstituentenstrukturbaum:**

$(N, Q, D, P, L)$

- N - endliche Menge von Knoten
- Q - endliche Menge von Etiketten
- D - schwache Teilordnung in  $N \times N$ , die Dominanzrelation
- P - starke Teilordnung in  $N \times N$ , die Präzedenzrelation
- L - Funktion von N in Q, die Etikettierfunktion

**Bedingungen:**

- Wurzelbedingung
- Exklusivitätsbedingung
- Kreuzungsfreiheit



## Äquivalente Ableitungen

$S \rightarrow NP VP$

$NP \rightarrow DET ADJ N$

$VP \rightarrow V$

S

NP VP

DET ADJ N VP

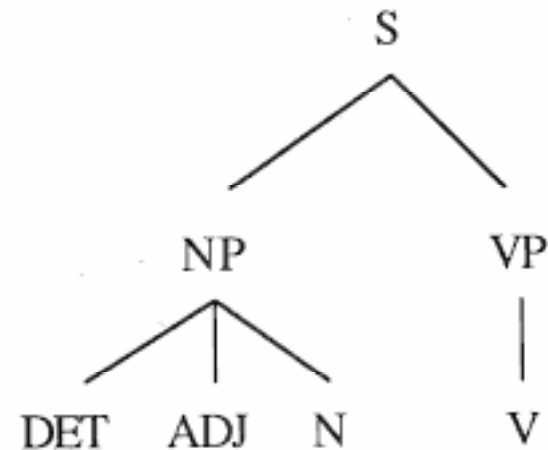
DET ADJ N V

S

NP VP

NP V

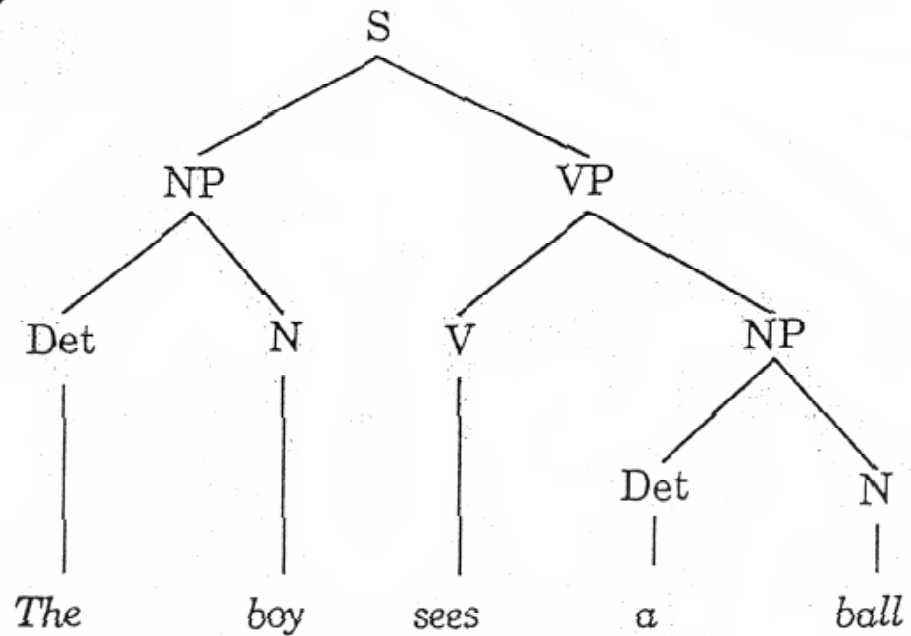
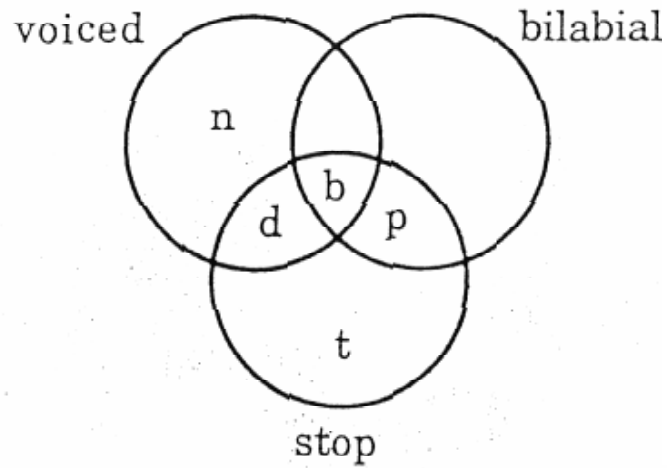
DET ADJ N V



# Zwei Arten von Strukturen



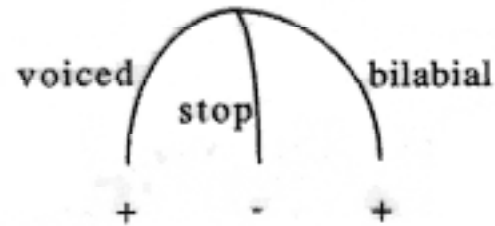
[voiced : -  
bilabial : +  
stop : +]



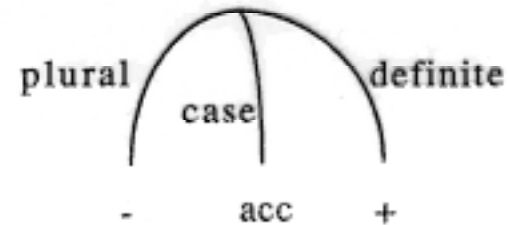
# Komplexe Merkmale I



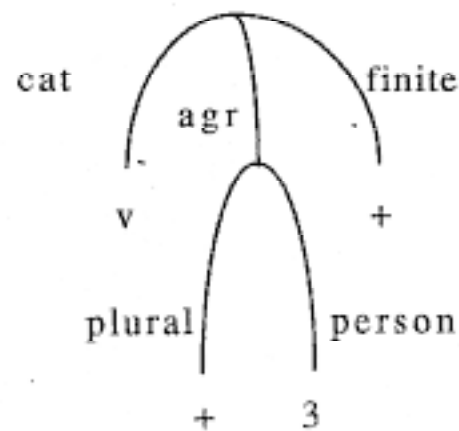
[voiced :-  
bilabial :+  
stop :+]



[plural :-  
definite :+  
case :acc]

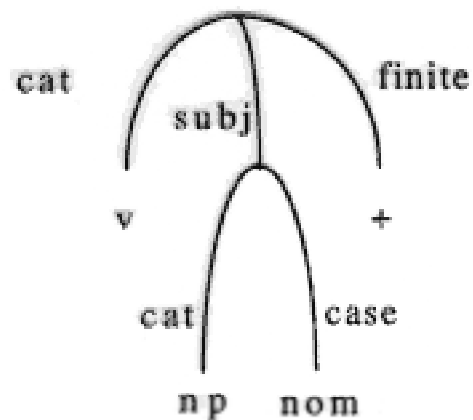


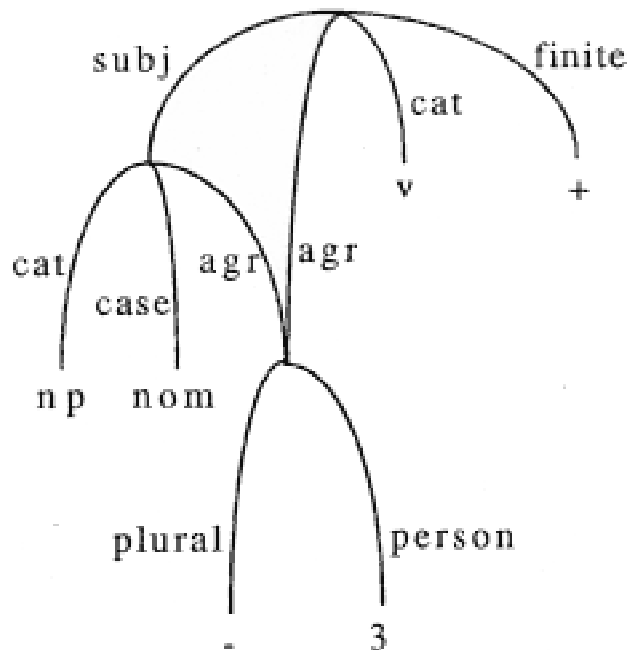
[cat :v  
finite :+  
agr : [plural :+  
          person :3]]



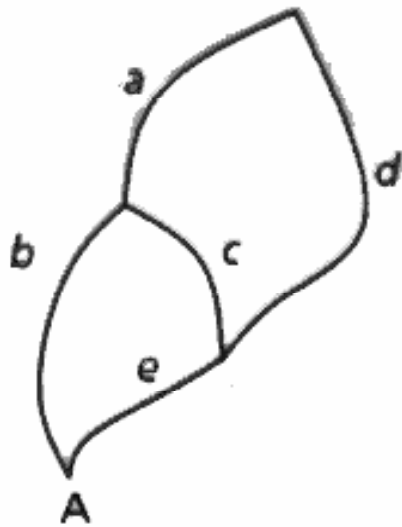
# Komplexe Merkmale II



$$\left[ \begin{array}{l} \text{cat} : \text{v} \\ \text{finite} : + \\ \text{subj} : \left[ \begin{array}{l} \text{cat} : \text{np} \\ \text{case} : \text{nom} \end{array} \right] \end{array} \right]$$


$$\left[ \begin{array}{l} \text{cat} : \text{v} \\ \text{finite} : + \\ \text{agr} : \left[ \begin{array}{l} \text{plural} : - \\ \text{person} : 3 \end{array} \right] \\ \text{subj} : \left[ \begin{array}{l} \text{cat} : \text{np} \\ \text{case} : \text{nom} \\ \text{agr} : \left[ \begin{array}{l} \text{plural} : - \\ \text{person} : 3 \end{array} \right] \end{array} \right] \end{array} \right]$$


# Drei Notationen für einen DAG



$$\left[ \begin{array}{l} a : \left[ \begin{array}{l} b : \boxed{1} A \\ c : \boxed{2} [e : \boxed{1}] \end{array} \right] \\ d : \boxed{2} \end{array} \right]$$

$$\langle a b \rangle = A$$

$$\langle a c e \rangle = \langle a b \rangle$$

$$\langle a c \rangle = \langle d \rangle$$

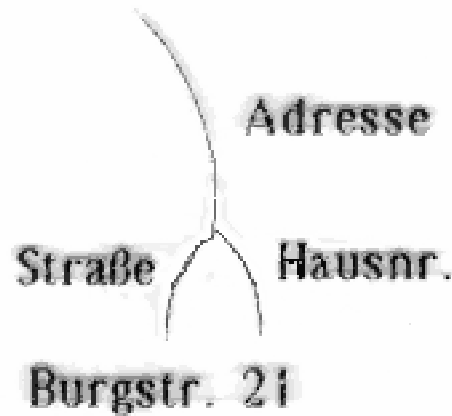
# Merkmalswerte



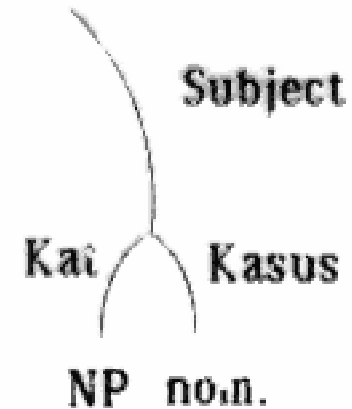
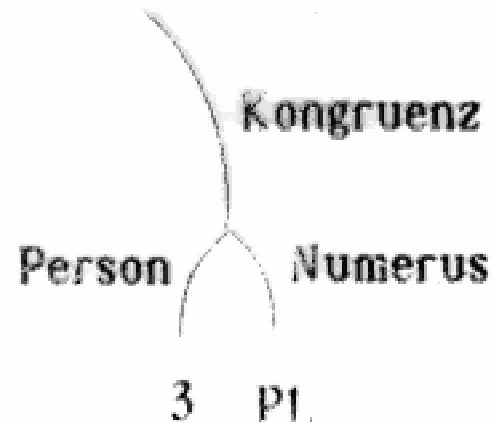
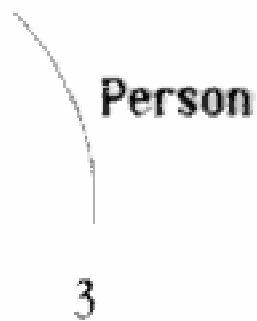
atomare Werte

komplexe Werte

komplexe Werte vom gleichen Typ



Vater

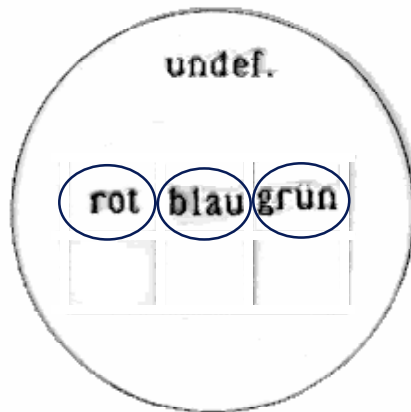




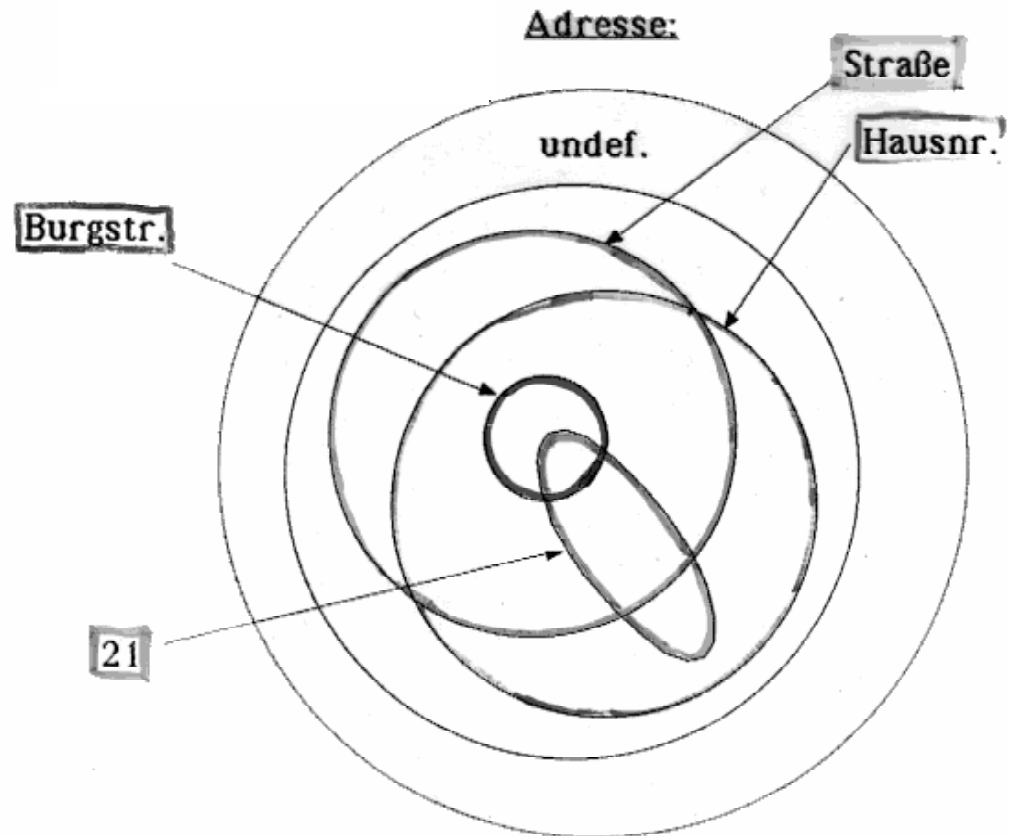
# Semantik der Merkmalsterme



**Farbe:**



**Adresse:**





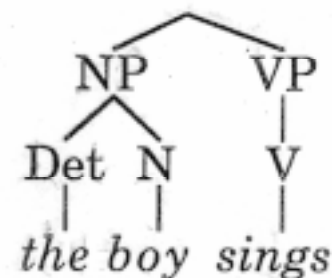
## Constituent Structure and Category Information

A traditional representation of a syntactic entity may be viewed as an ordered pair of a category and a constituent structure.  $\langle \text{cat}, \text{cs} \rangle \in \text{CAT} \times \text{CS}$

In a CF-PSG, *cat* is atomic and *cs* is a list of syntactic representations (a PS-tree without the root node).

$\langle \text{S},$   
   $\langle \text{NP},$   
     $\langle \text{Det},$   
       $\langle \text{the}, \emptyset \rangle \rangle,$   
     $\langle \text{N},$   
       $\langle \text{boy}, \emptyset \rangle \rangle \rangle,$   
   $\langle \text{VP},$   
     $\langle \text{V},$   
       $\langle \text{sings}, \emptyset \rangle \rangle \rangle \rangle$

*cat* = S  
*cs* =



In a simple unification grammar, *cat* is a feature structure and *cs* is a PS-tree.

# Phrasenstrukturregeln und Merkmalsstrukturen



$S \rightarrow NP VP$

$$\left[ \begin{array}{l} S: \left[ \begin{array}{l} \text{cat: S} \\ \text{finite: } \langle 1 \rangle \end{array} \right] \\ NP: \left[ \begin{array}{l} \text{cat: NP} \\ \text{agr: } \langle 2 \rangle \end{array} \right] \\ VP: \left[ \begin{array}{l} \text{cat: VP} \\ \text{agr: } \langle 2 \rangle \\ \text{finite: } \langle 1 \rangle \end{array} \right] \end{array} \right]$$

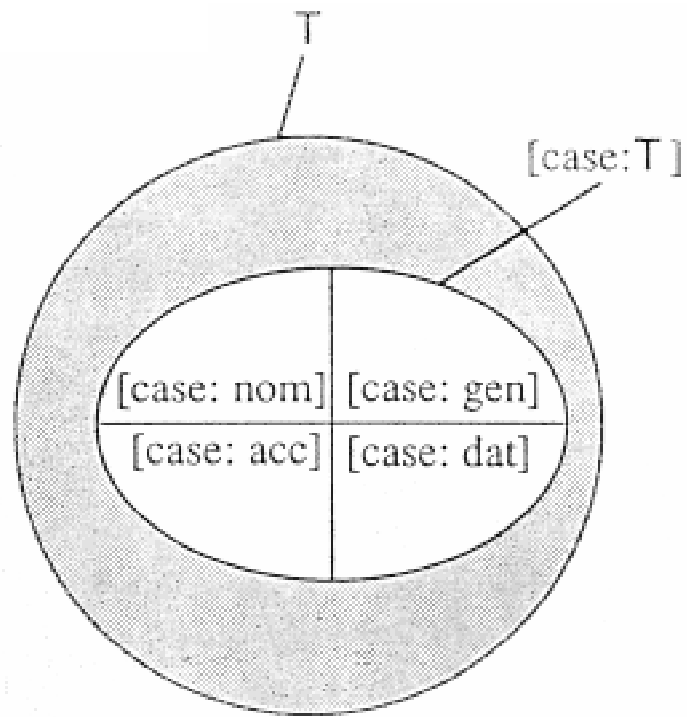
$X_0 \rightarrow X_1 X_2$

$$\left[ \begin{array}{l} X_0: \left[ \begin{array}{l} \text{cat: S} \\ \text{finite: } \langle 1 \rangle \end{array} \right] \\ X_1: \left[ \begin{array}{l} \text{cat: NP} \\ \text{agr: } \langle 2 \rangle \end{array} \right] \\ X_2: \left[ \begin{array}{l} \text{cat: VP} \\ \text{agr: } \langle 2 \rangle \\ \text{finite: } \langle 1 \rangle \end{array} \right] \end{array} \right]$$

# Merkmale als partielle Funktionen - Beispiel



[case:T] ≠ T



# Merkmale als totale Funktionen - Beispiel

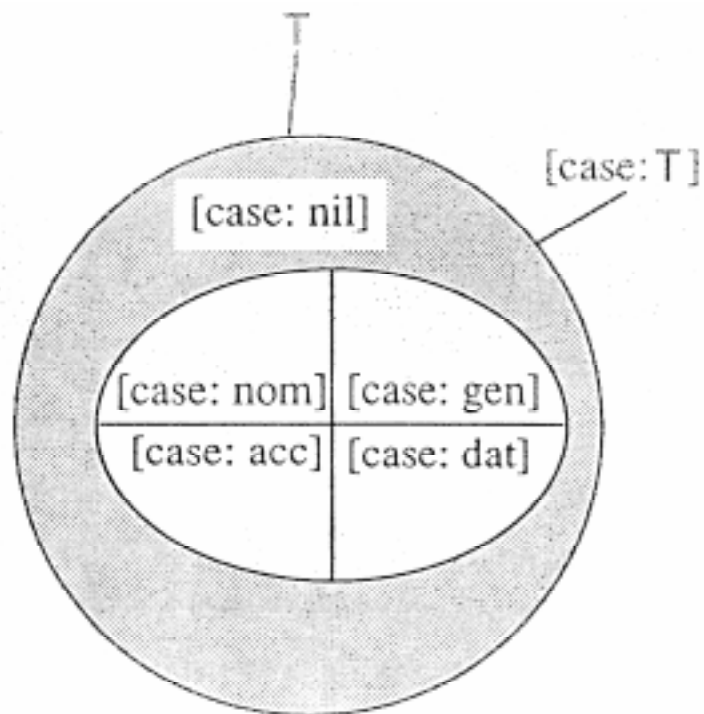


[case: nil]

[case: T] = T

corresponding to:

[case: ⊥] = ⊥



# Subsumption



Die Subsumptionsrelation ist eine schwache Teilordnung.

Daher ist die Relation:

transitiv

$$\forall t_1, t_2, t_3 [(t_1 \sqsubseteq t_2 \wedge t_2 \sqsubseteq t_3) \rightarrow t_1 \sqsubseteq t_3]$$

antisymmetrisch

$$\forall t_1, t_2 [(t_1 \sqsubseteq t_2 \wedge t_2 \sqsubseteq t_1) \rightarrow t_1 = t_2]$$

reflexiv

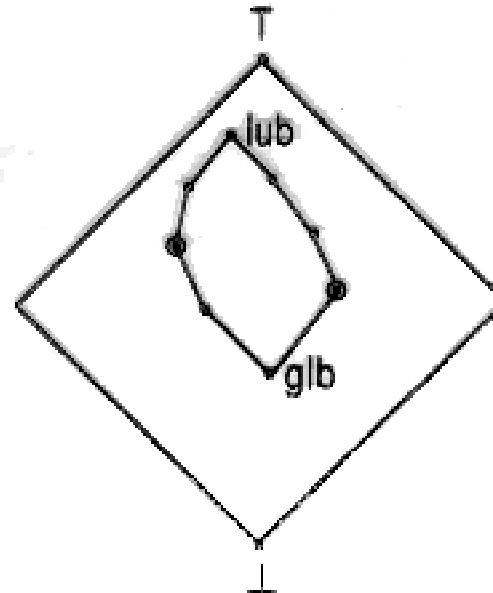
$$\forall t [t \sqsubseteq t]$$

In Unifikationsformalismen bildet die Subsumptionsrelation einen Verband oder einen Halbverband.

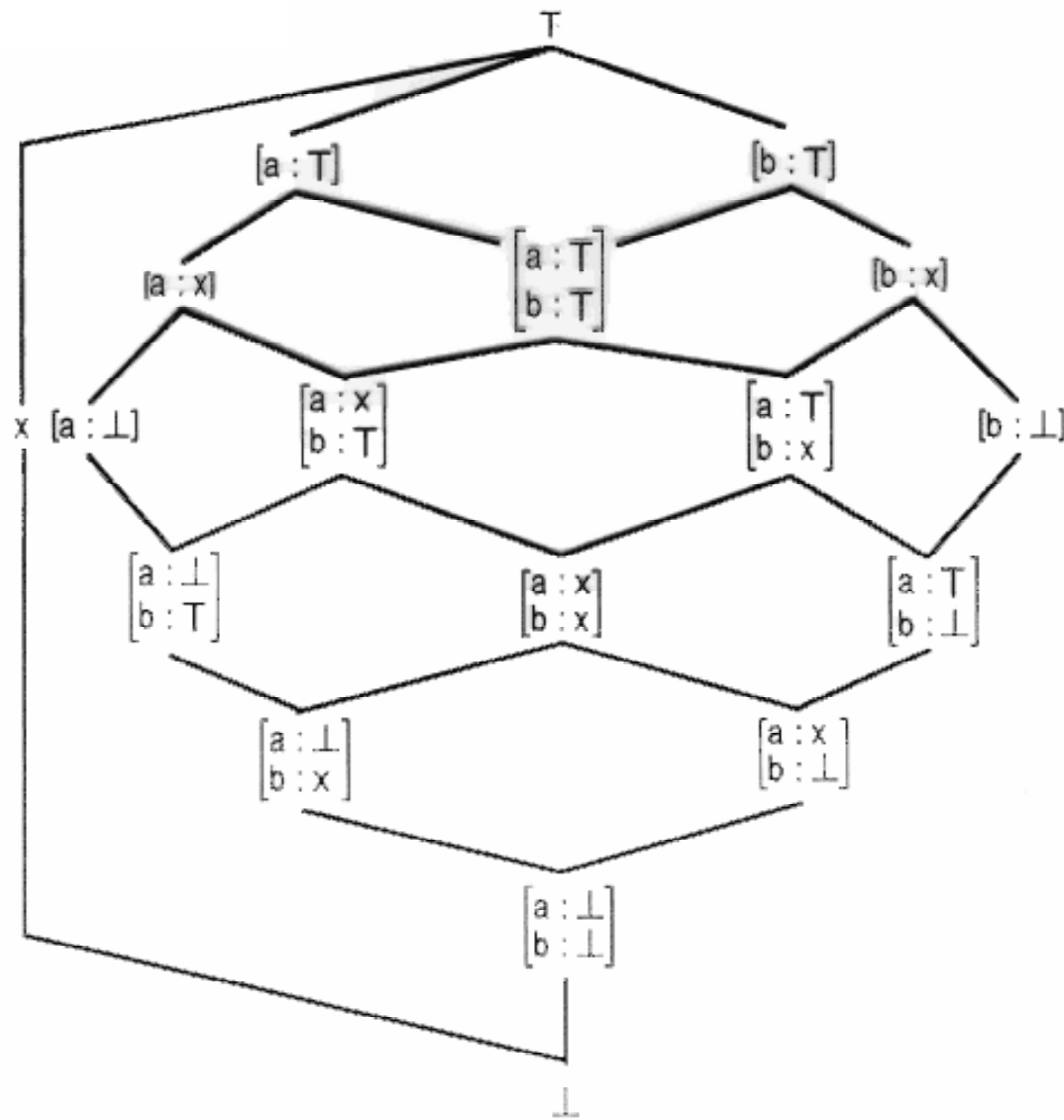
# Subsumptionsverband



glb: unification  
lub: generalization

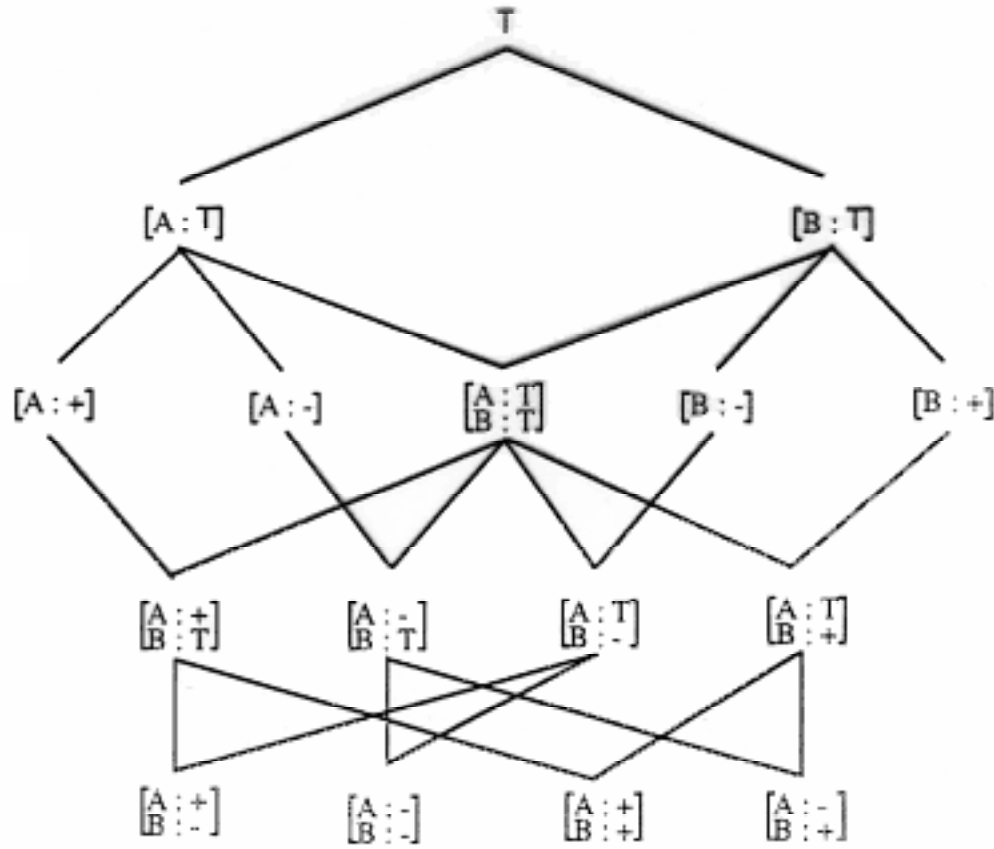


# Subsumptionsverband





# Subsumptionshierarchie





## Subsumption

$t1 \geq t2$  iff  $t2$  contains at least all the information contained in  
( $t2$  extends  $t1$ ,  $t2 \leq t1$ )

## Subsumption II



$T \geq \text{nom}$

$T \geq \perp$

$[\text{case: } T] \geq [\text{case: nom}]$

$\text{nom} \geq \text{nom}$

$\text{nom} \not\geq \text{acc}$

$[\text{tense: pres}] \not\geq \begin{bmatrix} \text{tense: } T \\ \text{agreement: } T \end{bmatrix}$

$[\text{tense: pres}] \geq \begin{bmatrix} \text{tense: pres} \\ \text{agreement: } T \end{bmatrix}$

$\begin{bmatrix} \text{subject: } [\text{case: } T] \\ \text{tense: pres} \end{bmatrix} \geq \begin{bmatrix} \text{subject: } [\text{case: nom}] \\ \text{tense: pres} \\ \text{agreement: } T \end{bmatrix}$

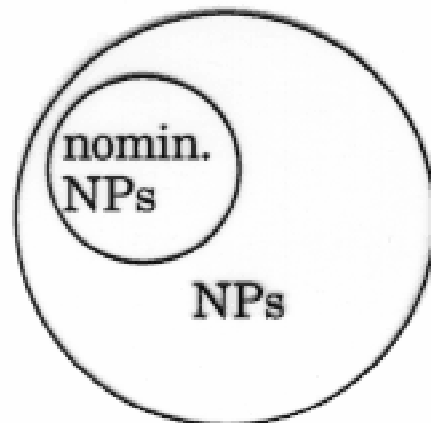


## Semantik der Subsumption:

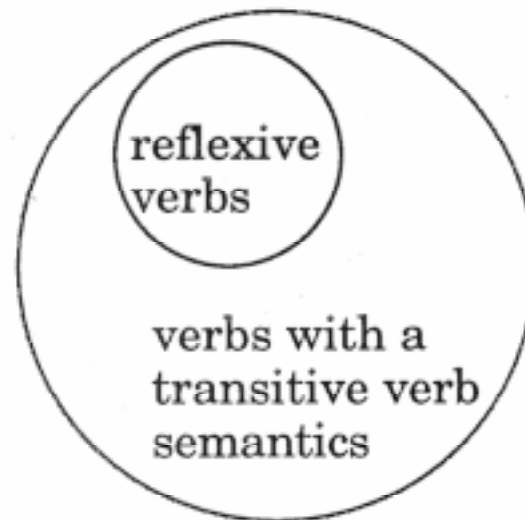
$$t_1 \sqsubseteq t_2 \leftrightarrow \llbracket t_1 \rrbracket \supseteq \llbracket t_2 \rrbracket$$

## Beispiele:

$$\begin{bmatrix} \text{cat: n} \\ \text{bar: 2} \end{bmatrix} \sqsubseteq \begin{bmatrix} \text{cat: n} \\ \text{bar: 2} \\ \text{case: nom} \end{bmatrix}$$



# Subsumption


$$\left[ \begin{array}{l} \text{cat: v} \\ \text{bar: 0} \\ \text{subject: [sem: T]} \\ \text{object: [sem: T]} \end{array} \right]$$
$$\left[ \begin{array}{l} \text{cat: v} \\ \text{bar: 0} \\ \text{subject: [sem: <1> T]} \\ \text{object: [sem: <1> T]} \end{array} \right]$$




## Idee:

Unifikation ist die Operation, die die Informationen zweier Merkmalsstrukturen vereint.

## Notation:

$t_1 \sqcup t_2$   
 $t_1 \wedge t_2$   
 $[t_1 \ t_2]$

## Definition:

Ein Typ  $t_0$  ist die Unifikation zweier Typen  $t_1$  und  $t_2$ , gdw.  $t_0$  der allgemeinste Typ ist, der sowohl von  $t_1$  als auch von  $t_2$  subsumiert wird.

## oder:

Ein Typ  $t_0$  ist die Unifikation zweier Typen  $t_1$  und  $t_2$ , gdw.  $t_0$  sowohl von  $t_1$  als auch von  $t_2$  subsumiert wird und wenn  $t_0$  alle anderen Typen  $t_i$  subsumiert die auch von  $t_1$  und  $t_2$  subsumiert werden.



# Eigenschaften der Unifikation

## Atome

$$(a_1 \sqcup a_2) = \perp \quad \text{gdw.} \quad a_1 \neq a_2$$

## top and bottom

$$(t \sqcup \top) = t$$

$$(t \sqcup \perp) = \perp$$

## Kommutativität:

$$(t_1 \sqcup t_2) = (t_2 \sqcup t_1)$$

## Assoziativität:

$$((t_1 \sqcup t_2) \sqcup t_3) = (t_1 \sqcup (t_2 \sqcup t_3))$$

## Idempotenz:

$$(t \sqcup t) = t$$



# Unifikation mit $\top$

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## Unifikation

### Unifikation mit $\top$

$$\top \sqcup t = t$$

### Unifikation zweier Atome

$$\alpha \sqcup \alpha = \alpha$$

$$\alpha \sqcup \beta = \text{FAIL oder } \perp \text{ gdw. } \alpha \neq \beta$$





# Unifikation mit $\top$

Unifikation eines Atoms mit einem komplexen Merkmalsterm

$$\alpha \sqcup \{a_1 : v_1, a_2 : v_2, \dots, a_n : v_n\} = \text{FAIL oder } \perp$$

Unifikation zweier komplexer Merkmalsterme  $t_1$  und  $t_2$

$$t_1 = \{\langle a_1, v_1 \rangle, \langle a_2, v_2 \rangle, \dots, \langle a_n, v_n \rangle\}$$

$$t_2 = \{\langle a_{n+1}, v_{n+1} \rangle, \langle a_{n+2}, v_{n+2} \rangle, \dots, \langle a_{n+m}, v_{n+m} \rangle\}$$

$$t_0 = t_1 \cup t_2$$

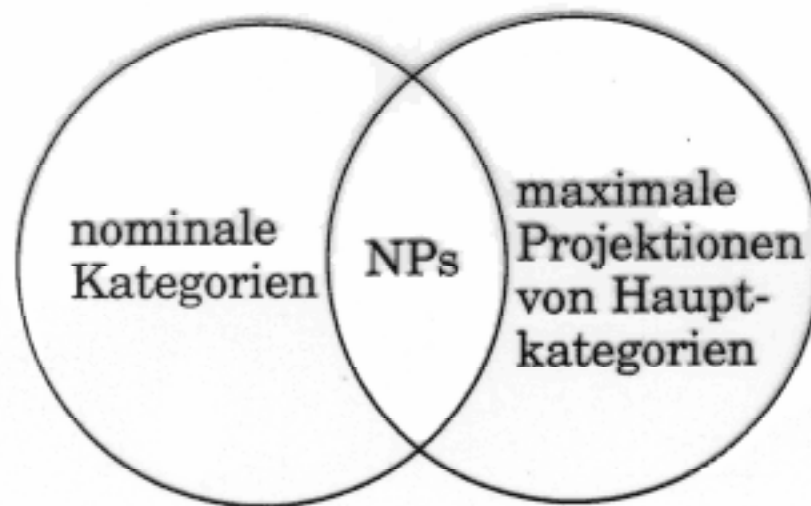
weil:

$$\begin{bmatrix} a_1 : v_1 \\ a_1 : v_2 \end{bmatrix} = [a_1 : (v_1 \sqcup v_2)]$$



$$\llbracket t_1 \cup t_2 \rrbracket = \llbracket t_1 \rrbracket \cap \llbracket t_2 \rrbracket$$

Beispiele:  $\llbracket \text{cat: n} \rrbracket \cup \llbracket \text{bar: 2} \rrbracket$




$$\left[ \begin{array}{l} \text{cat: v} \\ \text{bar: 0} \\ \text{agr: } \left[ \begin{array}{l} \text{per: 3} \\ \text{num: sg} \end{array} \right] \end{array} \right]$$

∪

$$\left[ \begin{array}{l} \text{cat: v} \\ \text{bar: 0} \\ \text{agr: } \left[ \begin{array}{l} \text{per: 1} \\ \text{num: sg} \end{array} \right] \end{array} \right]$$

3.Person  
Singular  
Verben

1.Person  
Singular  
Verben



## Semantik der Generalisierung

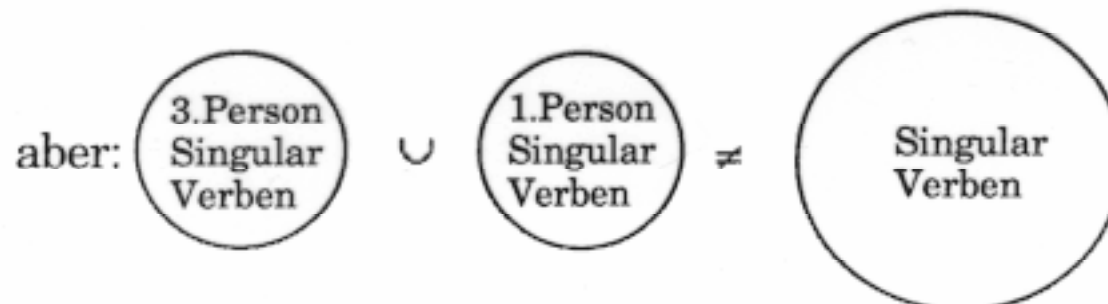
$$\llbracket t_1 \cap t_2 \rrbracket = \llbracket t_1 \rrbracket \cup \llbracket t_2 \rrbracket$$

Beispiel:

$$\left[ \begin{array}{l} \text{cat: v} \\ \text{bar: 0} \\ \text{agr: } \left[ \begin{array}{l} \text{per: 3} \\ \text{num: sg} \end{array} \right] \end{array} \right] \cup \left[ \begin{array}{l} \text{cat: v} \\ \text{bar: 0} \\ \text{agr: } \left[ \begin{array}{l} \text{per: 1} \\ \text{num: sg} \end{array} \right] \end{array} \right]$$

Generalisierung ohne Disjunktion:

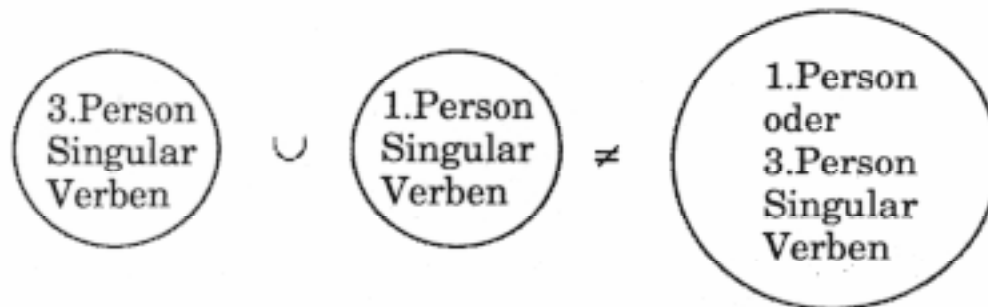
$$\left[ \begin{array}{l} \text{cat: v} \\ \text{bar: 0} \\ \text{agr: } \left[ \text{num: sg} \right] \end{array} \right]$$





Generalisierung als Disjunktion:

$$\left[ \begin{array}{l} \text{cat: v} \\ \text{bar: 0} \\ \text{agr: } \left[ \begin{array}{l} \text{per: } \left\{ \begin{array}{l} 1 \\ 3 \end{array} \right\} \\ \text{num: sg} \end{array} \right] \end{array} \right]$$



# Eigenschaften der Generalisierung/Disjunktion



## atoms

$(a_1 \sqcap a_2) = \top$  iff  $a_1 \neq a_2$  (with a flat atom lattice  
and without disjunction)

with disjunction:

$$(a_1 \sqcap a_2) = \begin{cases} a_1 \\ a_2 \end{cases}$$

## top and bottom

$$(t \sqcap \top) = \top$$

$$(t \sqcap \perp) = t$$

## commutativity:

$$(t_1 \sqcap t_2) = (t_2 \sqcap t_1)$$

## associativity:

$$((t_1 \sqcap t_2) \sqcap t_3) = (t_1 \sqcap (t_2 \sqcap t_3))$$

## idempotence:

$$(t \sqcap t) = t$$



## Idea:

Negation is the operation that expresses the complement of the information encoded in a feature structure.

## Notation:

$\neg t$

## Definition:

A type  $t_1$  is the negation of a type  $t_2$ , iff  $t_1$  is the generalization of all types whose unification with  $t_2$  is inconsistent.

## Negation II



### Some Equivalences Involving Negation:

$$\neg\neg t = t$$

$$\neg\perp = \top$$

$$\neg\top = \perp$$

$$t \cup \neg t = \perp$$

$$t \cap \neg t = \top$$

$$\neg(t_1 \cup t_2) = \neg t_1 \cap \neg t_2$$

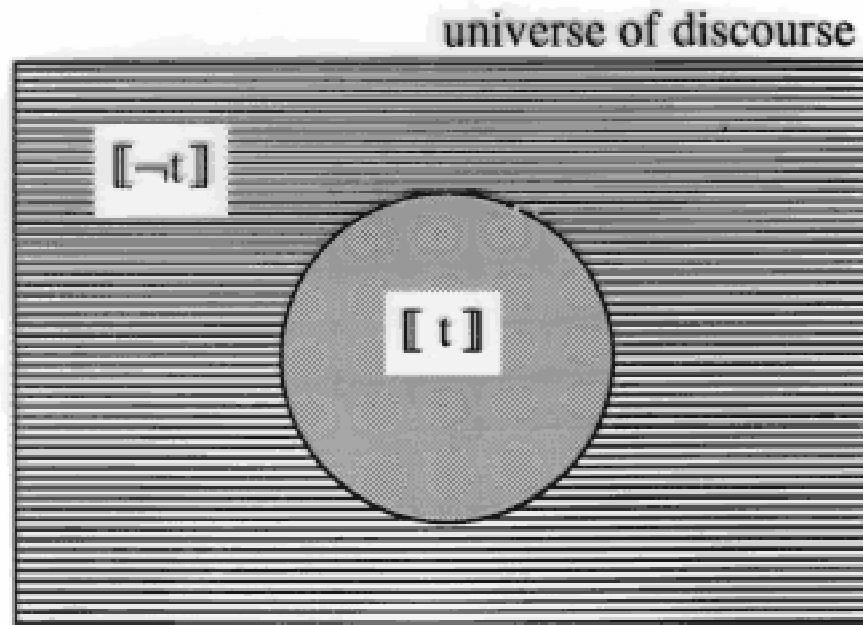
$$\neg(t_1 \cap t_2) = \neg t_1 \cup \neg t_2$$



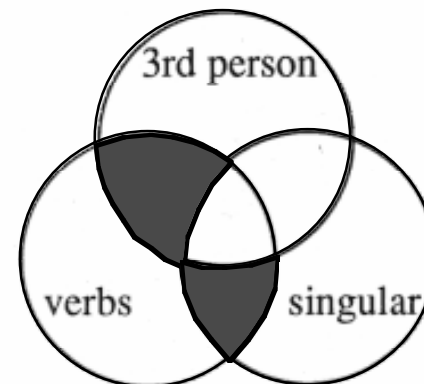
# Semantik der Negation



$$[[\neg t]] = \overline{[[t]]}$$



example:

$$\left[ \begin{array}{l} \text{cat: V} \\ \text{agr: } \left[ \begin{array}{l} \text{per: 3} \\ \text{num: sg} \end{array} \right] \end{array} \right]$$




# Implication

## Idea:

Implication is the operation that allows the statement of conditional feature types.

## Notation:

$$t_1 \rightarrow t_2$$
$$t_1 \supset t_2$$

## Definition:

The type  $t_1 \rightarrow t_2$  is the least informative type whose unification with  $t_1$  is subsumed by  $t_2$ .

# Implikation - Beispiel


$$\begin{bmatrix} \text{cat: V} \\ \text{finite: +} \end{bmatrix} \rightarrow [\text{tense: T}]$$

# Semantik der Implikation



$$\llbracket t_1 \rightarrow t_2 \rrbracket = \overline{\llbracket t_1 \rrbracket} \cup \llbracket t_2 \rrbracket$$

$$s_1 = \llbracket t_1 \rrbracket$$

$$s_2 = \llbracket t_2 \rrbracket$$

	$s_1$	$\overline{s_1}$
$s_2$		
$\overline{s_2}$		