Agnostic Possible Worlds Semantics

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Abstract. Working within standard classical higher-order logic, we propose a possible worlds semantics (PWS) which combines the simplicity of the familiar Montague semantics (MS), in which propositions are sets of worlds, with the fine-grainedness of the older but less well-known **tractarian** semantics (TS) of Wittgenstein and C.I. Lewis, wherein worlds are maximal consistent sets of propositions. The proposed **agnostic** PWS makes neither montagovian nor tractarian ontological commitments, but is consistent with (and easily extensible to) either alternative (among many others). It is technically straightforward and, we believe, capable of everything linguists need PWS to do, such as interfacing with a logical grammar and serving as a basis for dynamic semantics.

Keywords: propositions, possible worlds, maximal consistent sets, Montague semantics, tractarian semantics

1 Introduction

1.1 Montague Semantics

When Montague [1974] pioneered the systematic application of mathematical logic to the analysis of natural language meaning, he borrowed one key idea from Kripke [1963], and another from Carnap [1947]. From Kripke came the assumption of "an arbitrary set K of 'possible worlds' and a function $\Phi(P, H)$ assigning to each proposition [= atomic formula] P a truth-value in the world H", in contradistinction to the earlier notion of a possible world as a maximal consistent set of propositions (Wittgenstein [1921], C.I. Lewis [1923]) or formulas (Carnap [1947], Kripke [1959]). And from Carnap came the idea of a linguistic meaning as a Carnapian *intension*, a function whose domain is the set of possible worlds. In the case of a declarative sentence, the meaning—the proposition expressed by the sentence—is nothing more or less than (the characteristic function) of a set of these possible worlds, because Carnap followed Frege [1892] in assuming that the reference of a sentence is the truth value of the proposition that it expresses.

Together, these two ideas have as consequences that the set of propositions is a powerset algebra, the meanings of the English 'logic words' are just the familiar boolean operations on sets of worlds, and the centrally important notion of entailment is just the relation between them of subset inclusion. Because of its sheer simplicity and familiarity (skillfully illuminated in the influential textbook by Dowty et al. [1981]), and its early adoption by certain philosophers well-known to linguists, most notably Stalnaker [1976, 1984], this style of PWS became, and remains, *the* mainstream framework for theorizing about natural language meaning in the linguistic semantics community. At the same time, it has long been recognized—at least as early as C.I. Lewis [1943] and Carnap [1947]—that treating sentence meanings as sets of worlds also has the troubling consequence that entailment is *antisymmetric*, i.e. that distinct sentences with the same truth conditions *mean* the same thing, the best-known aspect of what is more generally known as the 'granularity problem' (that distinct linguistic expressions with necessarily identical denotations have the same meaning).

1.2 Structured Meanings

The (independent) responses of Lewis and Carnap to this challenge were, in essence, that the meanings of sentences are not the intensions associated with them, but rather 'structured' objects with lexical meanings as parts. Since then, one version or other of this general approach has been embraced by a number of philosophers (e.g. D. Lewis [1970], Cresswell [1985], Soames [1987], King [1996, 2007]). The various versions differ both with respect to what kinds of lexical meanings are embedded within the structures (e.g. whether they are intensional or extensional) and what kinds of structure they have (e.g. tuples, phrase markers, some version of Chomskian LF). And although structured-meaning approaches are broadly familiar to the linguistic semantics community, they seem not to have gained much of a foothold there, perhaps for lack of a canonical and accessible exposition with a degree of formal explicitness comparable to that of Dowty et al. [1981].

1.3 Propositions in Themselves

However, long before Carnap, Kripke, and Montague, there were well worked out conceptions of propositions as things in their own right (independent of sentences that might express them or conditions under which they might be true); and of worlds, not as unanalyzed primitives, but rather as certain sets of propositions (the *maximal consistent* ones). As early as 1837, Bolzano's notion of 'proposition in itself' (*Satz an sich*) embodied most of the key properties that present-day semanticists attribute to propositions:

a. They are expressed by declarative sentences.
b. They are the primary bearers of truth and falsity; a sentence is only secondarily, or derivatively, true or false, depending on what proposition it expresses.
c. They are the objects of the attitudes, i.e. they are the things that are known, believed, doubted, etc.
d. They are not linguistic.

e. They are not mental.

f. They are not located in space or time.

g. Sentences in different languages, or different sentences in the same language, can express the same proposition.

h. Two distinct propositions can entail each other.

1.4 Tractarian Semantics

Wittgenstein [1921] seems to have been the first to explicitly identify worlds with maximal consistent sets of facts, a characterization which we henceforth call **tractarian**. More precisely, the worlds of the *Tractatus* are maximal consistent assemblages of positive and negative facts. A fact (*Tatsache*), the closest counterpart in the *Tractatus* to our (or Bolzano's) propositions, consists of the (non)existence of a state of affairs (*Sachverhalt*); the term 'proposition' (*Satz*) is reserved for the linguistic entities that express (potential) facts, or equivalently, describe states of affairs.

More technically precise, but much less-known, is C.I. Lewis [1923], wherein a world is defined to be a 'system of facts' which is maximal in the sense of containing each fact or its 'contradictory'. In Lewis' terminology, 'facts' correspond to our propositions and the 'actual facts' of a system are those facts which belong to it ; while a 'system' is what would later come to be called a *proper filter* (a proper subset of the propositions, closed under conjunction and entailment). A significant advantage of Lewis' theory over Wittgenstein's is that it is not *atomistic*. That is, there is no requirement that there be a collection of 'basic facts', i.e. the ones expressed in the *Tractatus* system by elementary propositions (*Elementarsätze*).

More recent tractarian characterizations of possible worlds by philosophers include those of Adams [1974], Plantinga [1974], and Lycan [1979]. Within linguistics, there have been several logical theories of natural language meaning which take propositions to be primitives rather than sets of worlds, e.g. Thomason [1980], Muskens [2005], and Pollard [2008, 2011], the last of which is explicitly tractarian.

As is the case with structured meanings, none of the propositions-as-primitives approaches, tractarian or not, has attracted much of a following among semanticists, perhaps because the philosophers in question have not been as closely associated with the linguistic community, or perhaps for want of a sufficiently accessible exposition of the underlying technicalia. Although we strongly believe that this robust tradition, alternative to both montagovian PWS and structured-meaning approaches, deserves a fuller hearing in the linguistic community, it is not our purpose here to provide such an exposition. Instead, we will propose a *weakening* of MS, a PWS which we believe possesses all the positive attributes of MS but without the fatal identification of propositions with sets of worlds and the concomitant antisymmetry of entailment. This **agnostic** PWS is a weakening in the sense that, if the type p of propositions is identified with the type w $\rightarrow$ t of sets of worlds, then the addition of a single axiom produces a theory equivalent to Montague semantics (or, more precisely, to Gallin's ([1975]) reformulation of MS within the simple theory of types). On the other hand, by adding different axioms instead, we can obtain TS, or a wide range of alternative semantic theories. In practical terms, working semanticists can just use the weak theory 'off the shelf', without taking on further ontological commitments that don't have any empirical consequences.

2 The Theory

2.1 Preliminaries

We work within the simple theory of types of Church [1940] as modified by Henkin [1950] with the addition of the axiom of truth-value extensionality, identifying truth value equality with biimplication.

We adopt the following notational conventions. Application terms are written $(f\ a)$, not $f(a)$. Application associates to the left, so $(f\ a\ b)$ abbreviates $((f\ a)\ b)$. Outermost parentheses are often dropped. We abbreviate multiple abstractions, e.g. $\lambda_{xy}.a$ for $\lambda_x.\lambda_y.a$. Functional types are written $A \rightarrow B$; and implication associates to the right, so $A \rightarrow B \rightarrow C$ abbreviates $A \rightarrow (B \rightarrow C)$. Some binary function symbols are written infix without comment, e.g. p and q, p entails q. The turnstyle '$\vdash$' is used both for typing judgments of terms and for assertions of higher-order provability.

2.2 Types

Besides Church's types o and ι, here (following Montague) called e and t, we employ two additional basic types w (worlds) and p (propositions). We adopt the following type abbreviations:

a. $\mathrm{p}_{0} =_{\mathrm{def}} \mathrm{p}$
b. $\mathrm{p}_{n+1} =_{\mathrm{def}} \mathrm{e} \rightarrow \mathrm{p}_n$

2.3 Constants and Axioms

In MS, being true at is the relation between propositions (qua sets of worlds) and worlds such that p is true at w just in case $w \in p$; whereas in TS, it is the relation between propositions and worlds (*qua* maximal consistent sets of propositions) such that p is true at w just in case $p \in w$. In agnostic semantics, hereafter AS, by contrast, we only assume that being true at, denoted by the constant @, is *some relation or other* between propositions and worlds, without committing to whether it is membership, inverse membership, or something else altogether. Hence we have

$$\vdash @ : \mathrm{p} \rightarrow \mathrm{w} \rightarrow \mathrm{t} \text{ ('is true at')}$$

Axioms and definitions to be given below will then ensure that this relation has the two crucial properties that, for any world w and any propositions p and q:

a. the set of facts of w (i.e. the propositions which are true at w) form a maximal consistent set; and
b. p entails q iff q is true at every world where p is true.

We begin with the following abbreviations:

$\mathsf{facts} =_{\text{def}} \lambda_{wp}.p@w$
$\mathsf{entails} =_{\text{def}} \lambda_{pq}.\forall_w.p@w \rightarrow q@w$
$\equiv\; =_{\text{def}} \lambda_{pq}.(p \;\mathsf{entails}\; q) \wedge (q \;\mathsf{entails}\; p)$ ('equivalent to')

Thus $\mathsf{facts}\ w$ is the set of propositions true at w; p entails q just in case q is true at every world where p is; and p is (truth-conditionally) equivalent to q just in case p and q are mutually entailing.

The preceding three definitions could just as well have been written as axioms:

$\vdash \forall_w.(\mathsf{facts}\ w) = \lambda_p.p@w$
$\vdash \forall_{pq}.(p \;\mathsf{entails}\; q) \leftrightarrow \forall_w.p@w \rightarrow q@w$
$\vdash \forall_{pq}.(p \equiv q) \leftrightarrow (p \;\mathsf{entails}\; q) \wedge (q \;\mathsf{entails}\; p)$

Note that the last of these could be expressed equivalently as

$\vdash \forall_{pq}.(p \equiv q) \leftrightarrow \forall_w.(p@w) = (q@w)$

which will become relevant later when we generalize the notion of equivalence from propositions to meanings of other types.

Next, we introduce constants for the usual proposition-level connectives and quantifiers, as follows:

$\vdash \mathsf{truth} : \mathrm{p}$ (a necessary truth)
$\vdash \mathsf{falsity} : \mathrm{p}$ (a necessary falsehood)
$\vdash \mathsf{not} : \mathrm{p} \rightarrow \mathrm{p}$ (propositional negation)
$\vdash \mathsf{and} : \mathrm{p} \rightarrow \mathrm{p} \rightarrow \mathrm{p}$ (propositional conjunction)
$\vdash \mathsf{or} : \mathrm{p} \rightarrow \mathrm{p} \rightarrow \mathrm{p}$ (propositional disjunction)
$\vdash \mathsf{implies} : \mathrm{p} \rightarrow \mathrm{p} \rightarrow \mathrm{p}$ (propositional implication)
$\vdash \mathsf{forall} : (\mathrm{e} \rightarrow \mathrm{p}) \rightarrow \mathrm{p}$ (propositional universal)
$\vdash \mathsf{exists} : (\mathrm{e} \rightarrow \mathrm{p}) \rightarrow \mathrm{p}$ (propositional existential)

Of these, the connectives not, and , or , and implies would be employed in a (static) semantically interpreted grammar as translations, respectively, for the sentential negation *it is not the case that*, sentential conjunction *and*, sentential disjunction *or*, and sentence subordinator *if*; and the quantifiers would be employed as expected in the translations of the quantification determiners *every* and *some*:

$\mathsf{every} =_{\text{def}} \lambda_{PQ}.\mathsf{forall}\ \lambda_x.(P\ x)\ \mathsf{implies}\ (Q\ x)$
$\mathsf{some} =_{\text{def}} \lambda_{PQ}.\mathsf{exists}\ \lambda_x.(P\ x)\ \mathsf{and}\ (Q\ x)$

The propositional connectives and quantifiers are subject to the following axioms:

$\vdash \forall_w.\mathsf{truth}@w$

$\vdash \forall_w.\neg(\mathsf{falsity}@w)$

$\vdash \forall_{pw}.(\mathsf{not}\ p)@w \leftrightarrow \neg(p@w)$

$\vdash \forall_{pqw}.(p\ \mathsf{and}\ q)@w \leftrightarrow (p@w \wedge q@w)$

$\vdash \forall_{pqw}.(p\ \mathsf{or}\ q)@w \leftrightarrow (p@w \vee q@w)$

$\vdash \forall_{pqw}.(p\ \mathsf{implies}\ q)@w \leftrightarrow (p@w \to q@w)$

$\vdash \forall_{Pw}.(\mathsf{forall}\ P)@w \leftrightarrow \forall_x.(P\ x)@w$

$\vdash \forall_{Pw}.(\mathsf{exists}\ P)@w \leftrightarrow \exists_x.(P\ x)@w$

The first six of these axioms say that, in an interpretation, the propositions form a *pre-boolean algebra*, i.e. an algebra that satisfies all the boolean axioms up to the equivalence relation induced by the underlying preorder; in the present case, that preorder is entailment and the induced equivalence is truth-conditional equivalence. Crucially, nothing licenses the inference that entailment is antisymmetric, and so the granularity problem does not arise. Of course there is no shortage in the literature of solutions to the granularity problem, but we believe that this one is by far the simplest.

The Facts of a World are a Maximal Consistent Set

In a (pre-)boolean algebra, a **maximal consistent set**, also called an **ultrafilter**, is a subset S which 'settles all issues' in the sense that for any algebra element p, either p or its complement is in S, but not both; and which is upper-closed relative to the underlying (pre-)order and closed under the meet operation of the algebra.

In Lewis' [1923] formulation of TS, worlds are easily seen to be maximal consistent by definition, once we align his terminology with more a contemporary one. Specifically: Lewis' 'systems' correspond to 'proper filters'; 'facts' to 'propositions'; 'actual facts' to 'facts' *simplicter*; 'requires' to 'entails', the 'joint fact' of two facts to their 'propositional conjunction'; the 'contradictory of a fact' to its 'propositional negation'; and 'p is inconsistent with q' to 'p entails $\mathsf{not}\ q$'.

In MS, the facts of a world are maximal consistent because the boolean preorder of propositions is the powerset of the set of worlds with entailment as subset inclusion and propositional conjunction as intersection, so that the set of facts of a world w is just the (principal) ultrafilter consisting of all the propositions which have w as a member.

In AS, that the set of facts of any world is maximal consistent is an (easy) theorem of the axioms and definitions given in the previous subsection. To prove it, we just define maximal consistency within our semantic theory in the obvious way (here the variable s is of type $\mathrm{p} \to \mathrm{t}$):

$\mathsf{upc} =_{\text{def}} \lambda_s.\forall_{pq}.((s\ p) \wedge (p\ \mathsf{entails}\ q)) \rightarrow (s\ q)$ ('upper-closed')
$\mathsf{cjc} =_{\text{def}} \lambda_s.\forall_{pq}.((s\ p) \wedge (s\ q)) \rightarrow (s\ (p\ \mathsf{and}\ q))$ ('conjunction-closed')
$\mathsf{sai} =_{\text{def}} \lambda_s.\forall_p.((s\ p) \vee (s\ (\mathsf{not}\ p)) \wedge \neg((s\ p) \wedge (s\ (\mathsf{not}\ p)))$ ('settles all issues')
$\mathsf{mxc} =_{\text{def}} \lambda_s.(\mathsf{upc}\ s) \wedge (\mathsf{cjc}\ s) \wedge (\mathsf{sai}\ s)$ ('maximal consistent')

Then the theorem in question takes the form:

$$\vdash \forall_w.\mathsf{mxc}\ (\mathsf{facts}\ w)$$

3 From Agnostic Semantics to Montagovian Semantics

Among fine-grained approaches to natural language semantics, AS is perhaps unique in being *consistent with MS*. In fact, AS *becomes* MS (or more precisely, becomes Gallin's [1975] reformulation of MS within the simple theory of types) with the addition of the following **montagovian axiom:**

$\vdash \forall_{pw}.p@w = (p\ w)$

or, equivalently: $@ =_{\text{def}} \lambda_p.p$. In order for this to be well-typed, we must concomitantly drop the assumption that p is a basic type and instead treat it as an abbreviation for w → t, so that *every set of worlds is a proposition.* It is easily verified that, with this addition, each proposition is identical with its own set of facts:

$\vdash \forall_p.p = \lambda_w.p@w$

that entailment reduces to subset inclusion, and that the propositional connectives become identified with the usual set-theoretic operations on the powerset of the set of worlds. Other consequences include the following:

a. Entailment is antisymmnmetric, i.e. equivalent propositions are identical.
b. For every set of propositions, there is a proposition which, necessarily, is true iff every member of the set is true (namely, the intersection of the set.)
c. For every world w, there is a proposition true only at that world (namely the singleton set whose member is w).
d. Every world is uniquely determined by its set of facts.
e. Not every maximal consistent set of propositions is the set of facts for some world.

The first two of these consequences are explicitly defended by Stalknaker [1976, 1984]. But it appears to us that, since they are inescapable consequences of his assumption of the montagovian axiom, he is just making a virtue of necessity. Consequence (c), though evidently lacking empirical consequences, is perhaps an ontological commitment that not every semanticist would wish to take on. Consequence (d) is, as Kripke [1963] notes, a property of the modal semantics in Kripke [1959] which he now wished to avoid, though again it may not have empirical consequences for natural language semantics. Consequence (e) arises because, for any world w, $(\mathsf{facts}\ w)$ is the *principal* filter over the powerset of the set of worlds (ordered by subset inclusion) generated by the singleton of w. But (assuming Choice), every infinite boolean algebra has a nonprincipal ultrafilter. This leaves MS in the rather uncomfortable position of being charged with providing an explanation for the fact that there are maximal consistent sets of propositions which don't correspond to any possible world.

To summarize: there is nothing in AS for an advocate of MS to take issue with, since AS is a weaker theory than MS. Any semanticist or philosopher who is comfortable with the additional consequences and commitments of MS is free to add the montagovian axiom. From our perspective though, this is a rather perverse thing to do. To put it in terms of a metaphor: AS is a house that we invite semanticists to inhabit. Some might hesitate to accept, saying: but we are used to living in a house with a leaky roof, and this roof doesn't leak! To them, we say: fine, you can punch a hole in the roof. The montagovian axioms is the tool provided expressly for that purpose.

4 From Agnostic Semantics to Tractarian Semantics

The central tractarian tenet, that worlds are maximal consistent sets of propositions, is hard to formulate in standard higher-order logic. Intuitively, we would like to identify each world with its set of facts:

$$\vdash \forall_w . w = (\mathsf{facts}\ w)$$

Alas, this is ill-typed unless $\mathrm{w} = \mathrm{p} \rightarrow \mathrm{t}$. And we can't just identify w with $\mathrm{p} \rightarrow \mathrm{t}$ because not every set of propositions is a set of worlds, only the maximal consistent ones.[1] What we can do, however, is to assert that there is a *bijection* between worlds and maximal consistent sets of propositions. This we do with two axioms, for injectivity and surjectivity respectively. The first of these can be thought of as a **weak tractarian axiom**:

$$\vdash \forall_{vw} . ((\mathsf{facts}\ v) = (\mathsf{facts}\ w)) \rightarrow v = w$$

i.e. a world is *uniquely determined* by its set of facts (which, recall, has been shown to be maximal consistent).[2]

The other axiom, for surjectivity, says that *every* maximal consistent set of propositions is the set of facts for some world:

[1] This technical obstacle can be overcome by working in a version of higher-order logic with separation-style subtyping, such as the categorical logic of Lambek and Scott [1986]: then we identify w with the *subtype* of $\mathrm{p} \rightarrow \mathrm{t}$ whose characteristic function is denoted by mxc.

[2] Again, this is the ontological commitment that Kripke explicitly rejected in his [1963] semantics for normal modal logic (but not in his [1959] semantics for S5).

$$\vdash \forall_s.(\mathsf{mxc}\ s) \rightarrow \exists_w.s = \mathsf{facts}\ w$$

Together, these two axioms give a (strongly) TS essentially the same as Pollard's ([2008, 2011]) hyperintensional semantics (but expressed in standard HOL rather than categorical logic).

Unlike the montagovian extension of agnostic semantics, the (weak or strong) tractarian extensions are free of the (in our view pernicious) consequence that entailment is antisymmetric.[3] Of course, this is already true of agnostic semantics. At this point, it remains unclear to us whether adopting either of the tractarian ontological commitments (injectivity and surjectivity of the facts function) confers any empirical or theoretical advantages on the working semanticist. In the mean time, it seems that we can just go about our usual semantic business in the agnostic setting.

5 Business as Usual in Agnostic Semantics

Here we provide a few illustrations of how to conduct the normal daily business of Montague semantics in the absence of either montagovian or tractarian assumptions.

5.1 Word Meanings

As usual, we introduce lots of constants for word meanings, e.g.

$\vdash \mathsf{p} : \mathrm{e}$ (Pedro)
$\vdash \mathsf{c} : \mathrm{e}$ (Chiquita)
$\vdash \mathsf{m} : \mathrm{e}$ (Maria)
$\vdash \mathsf{donkey} : \mathrm{p}_{\mathit{1}}$
$\vdash \mathsf{farmer} : \mathrm{p}_{\mathit{1}}$
$\vdash \mathsf{yell} : \mathrm{p}_{\mathit{1}}$
$\vdash \mathsf{kick} : \mathrm{p}_{\mathit{2}}$
$\vdash \mathsf{give} : \mathrm{p}_{\mathit{3}}$
$\vdash \mathsf{believe} : \mathrm{e} \rightarrow \mathrm{p} \rightarrow \mathrm{p}$
$\vdash \mathsf{persuade} : \mathrm{e} \rightarrow \mathrm{e} \rightarrow \mathrm{p} \rightarrow \mathrm{p}$
$\vdash \mathsf{that} : \mathrm{p}_{\mathit{1}} \rightarrow \mathrm{p}_{\mathit{1}} \rightarrow \mathrm{p}_{\mathit{1}}$ (property conjunction)
$\vdash \mathsf{every} : \mathrm{p}_{\mathit{1}} \rightarrow \mathrm{p}_{\mathit{1}} \rightarrow \mathrm{p}$ (universal determiner)
$\vdash \mathsf{some} : \mathrm{p}_{\mathit{1}} \rightarrow \mathrm{p}_{\mathit{1}} \rightarrow \mathrm{p}$ (existential determiner)

These can be made subject to nonlogical axioms (cf. Montague's 'meaning postulates' , or equivalently, be treated as abbreviations, e.g.:

$\mathsf{that} =_{\mathrm{def}} \lambda_{PQx}.(P\ x)\ \mathsf{and}\ (Q\ x)$
$\mathsf{some} =_{\mathrm{def}} \lambda_{PQ}.\mathsf{exists}(\lambda_x.(P\ x)\ \mathsf{and}\ (Q\ x)) = \lambda_{PQ}.\mathsf{exists}(P\ \mathsf{that}\ Q)$
$\mathsf{every} =_{\mathrm{def}} \lambda_{PQ}.\mathsf{forall}(\lambda_x.(P\ x)\ \mathsf{implies}\ (Q\ x))$

[3] The framework of Jónsson and Tarski [1951] is a tractarian system with antisymmetry of entailment, as it is an elaboration (with the addition of boolean operators) of Stone's [1936] duality theory of boolean (not pre-boolean) algebras.

5.2 Extensions of Meanings

We recursively define the set of **meaning types** as follows:

a. e is a **basic** meaning type.
b. p is a **basic** meaning type.
c. If A and B are meaning types, then $A \to B$ is a **functional** meaning type.
c. Nothing else is a meaning type.

For each meaning type A, there is a corresponding type $\mathrm{Ext}(A)$ for the extensions of meanings of type A.

a. $\mathrm{Ext}(\mathrm{e}) = \mathrm{e}$
b. $\mathrm{Ext}(\mathrm{p}) = \mathrm{t}$
c. $\mathrm{Ext}(A \to B) = A \to \mathrm{Ext}(B)$ (*not* $\mathrm{Ext}(A) \to \mathrm{Ext}(B)$)

To handle the notion of the **extension** of a meaning at a world, we extend the @ function to all meaning types by introducing a family of constants

$$\vdash @_A \ : A \to \mathrm{w} \to \mathrm{Ext}(A)$$

where A ranges over meaning types. (Usually the type subscript on '@' is omitted.) Here $a@w$ is read 'the extension of a at w'. The @ functions are subject to the axioms:

$\vdash \forall_{xw}.x@_\mathrm{e}w = x$
$\vdash \forall_{pw}.p@_p w = p@w$
$\vdash \forall_{fw}.f@w = \lambda_x.(f\ x)@w$ (A a functional type)

The first of these embodies a version of the direct referential theory of names: that the meaning of a name coincides with its reference at whatever world. The second captures the Fregean identification of the reference of a sentence with the truth value of the proposition it expresses. The third, the interesting one, defines the extension of a function meaning in terms of the extensions of the values of the function for all possible arguments. To take a very simple example: to say that Pedro is a farmer in the actual world $\mathbf{w}_0$ is to say that $\mathsf{farmer}@\mathbf{w}_0\ \mathsf{p}$, which in turn amounts to $(\mathsf{farmer\ p})@\mathbf{w}_0$. That is: to be a $\mathbf{w}_0$-farmer is no more and no less than being an entity such that the proposition that that entity is a farmer is one of the facts of $\mathbf{w}_0$.

5.3 Equivalence of Meanings, Generalized

Recall that two propositions are **equivalent** iff they are true at the same worlds, i.e. $p \equiv q$ iff for every world w, $p@w = q@w$. More generally, we can now say that two meanings a and b of the same type are **equivalent** iff, for every world w, a and b have the same extension at w. That is, for *every* meaning type A, we define **meaning equivalence** by:

$$\equiv_A =_{\text{def}} \lambda_{xy}.\forall_w.x@w = y@w$$

As with mutually entailing propositions, nothing forces equivalent functional meanings to be equal. In the presence of the montagavian axiom, however, equivalence of functional meanings reduces to identity.

6 Conclusion

We sketched the outlines of a weak PWS, of which the familiar Montague semantics and the older, largely forgotten, PWS of Wittgenstein and C.I. Lewis can be viewed as straightforward extensions. We suggested the possibility that the core 'agnostic' theory might replace Montague semantics as a practical framework for the analysis of linguistic meaning. Among the work that remains to be done is a more detailed formalization of the various schemes of PWS proposed by philosophers and linguists; a consideration of the status of structured-meaning theories; and development of a more robust agnostic fragment (covering, for example, interrogatives, conditionals, and various categories of projective meaning). Some of these tasks are taken up in Plummer and Pollard [in prep.].

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