Advances in Logical Grammar: Semantics Basics for Syntacticians

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Expressions, Utterances, and Meanings (1/2)

- We distinguish **expressions** from **utterances** (uses of expressions in specific circumstances).
- Each utterance has (or **expresses**) a **meaning**, which is jointly determined by:
  - what expression the utterance is a use of
  - certain aspects of the circumstances.

Expressions, Utterances, and Meanings (2/2)

Meanings are external to language and to the minds of language users (though perhaps they can be mentally represented). For example:

- Meanings of declarative sentence utterances are **propositions**. (We'll discuss these in detail soon.)
- Meanings of proper noun utterances are **entities**. (This position is controversial, but we'll adopt it.)
- meanings of intransitive verb or common noun utterances are **properties**, usually (and here) analyzed as functions from entities to propositions.

Interdependence of Context and Utterance Meaning

- Those aspects of the circumstances of an utterance involved in the determination of its meaning are called its **context**.
- For example, what entity is expressed by a use of the name *Kim* depends on the context.
- Likewise, what proposition is expressed by a use of the declarative sentence *she kicked him* depends on the context.

- Conversely, each utterance helps create the context involved in determining the meaning of the *next* utterance:

  a. He sat down. A farmer walked in carrying a duck.

  b. A farmer walked in carrying a duck. He sat down.

Dynamic and Static Semantic Theories (1/2)

- This interdependence between context and utterance meaning is called **dynamicity**, and semantic theories that take dynamicity into account are called **dynamic**.
- Dynamicity plays a central rule in (for example) anaphora, (in-)definiteness, presupposition, conventional implicature, contrast, topicality, focus, and the relationship between questions and answers.
- Dynamic theories must formally model contexts.

Dynamic and Static Semantic Theories (2/2)

- Semantic theories that steer clear of dynamicity, by ignoring context or pretending that the context is held fixed, are called **static**.
- Usually (and here), dynamic semantic theories are built on the foundation of a static theory.
- As long as we are ignoring context, the distinction between expression and utterance is not so important, and we will not always make it terminologically.

Meaning and Extension

- We distinguish between a meaning and its **extension**.
  - The extension of a proposition is its truth value.
  - The extension of a property is (the characteristic function of) the set of things that have that property.
  - The extension of an entity is the entity itself.
  - There's a system to this, which we'll come to soon.
- What extension a meaning has can depend on **contingent fact**, or, informally, on how things are.

Reference

The **reference** of an (utterance of an) expression is the extension of its meaning, so this too can depend on how things are. For example:

- The reference of a declarative sentence is the truth value of the proposition it expresses.

- the reference of an intransitive verb or common noun is (the characteristic function of) the set of entities that have the property it expresses.

- the reference of a proper noun is the same as the entity it expresses.

Possible Worlds

- Most semantic theories take explicit account of the way that extensions (and therefore reference) can depend on how things are, or might be.
- Ways that things are or might be are called **(possible) worlds**, or just **worlds**.
- So a semantic theory that take these into account is called a **possible worlds semantics**.
- By a world, we mean not just a snapshot at a particular time, but a whole history, stretching as far back and as far forward as things go.
- One of the worlds, called the **actual** world, or just **actuality**, is the way things *really* are (again, stretching as far back and as far forward as things go).

Ways of Conceptualizing Worlds

There are different ways of conceptualizing worlds.

- In **tractarian** theories (named after Wittgenstein's (1918) *Tractatus Logico-Philosophicus*), worlds are certain sets of propositions, namely the *maximal consistent ones*. (Examples: Wittgenstein, C.I. Lewis, Robert Adams, Alvin Plantinga, William Lycan)
- In **kripkean** theories (based on Kripke's (1963) semantics of modal logic), worlds are taken to be theoretical primitives.
- **Montagovian** theories are kripkean theories in which propositions are defined to be (characteristic functions of) sets of worlds. (Examples: Richard Montague, David Kaplan, David Lewis, Robert Stalnaker)

Agnostic Possible Worlds Semantics

The **agnostic** possible worlds semantics used in this course is *neutral* among all these positions: it could be extended into either a tractarian or a kripkean (including montagovian) theory.

The Extension of a Meaning at a World

- We don't speak of a meaning as simply *having* a certain extension, but rather as having that extension *at a given world.*
- In particular, we don't speak of a proposition as simply being true or false, but rather as being true or false *at a given world.*
- In other words, we assume there is a relation between propositions and worlds, called **being true at**, and we say p is true at w (written $p@w$) if the ordered pair $\langle p, w\rangle$ is in this relation.
- As we'll see, for *any* meaning m, the extension of m at a world w can be defined in terms of the @ relation.

The Reference of an Expression at a World

- When we say that (an utterance of) an expression has reference r at a given world, what we mean is that the meaning it expresses has extension r at that world.
- In particular, when we say that (an utterance of) a sentence is true (or false) ate a given world, what we mean is that the proposition it expresses has extension true (or false) at that world.

Entailment (1/2)

- For two propositions p and q, we say p **entails** q provided, no matter how things are, if p is true when things are that way, then so is q.
- In terms of possible worlds semantics: p entails q if and only if, for every world w, if p is true at w, then so is q.
- Obviously entailment is a preorder (relexive and transitive).
- Two propositions are called (**truth-conditionally**) **equivalent** if they entail each other.
- Equivalence is obviously an equivalence relation (reflexive, transitive, and symmetric).

Entailment (2/2)

- As with truth (at a world), the use of the terms 'entailment' is extended from propositions to the (utterances of) declarative sentences that express them. (And likewise for 'equivalent'.)
- So 'S$_{1}$ entails S$_{2}$' means that the proposition expressed by S$_{1}$ entails the proposition expressed by S$_{2}$.

- Native speaker judgments about entailments between sentences (or better, in-context utterances of sentences) are important (some would say, the most important) data in testing semantic hypotheses.

Bolzano's Notion of Proposition (1/2)

- Something similar to the notion of proposition used here was first suggested by the mathematician/philosopher Bernard Bolzano (*Wissenschaftslehre*, 1837)—his term was *Satz an sich* 'proposition in itself':
- They are expressed by declarative sentences.
- They are the 'primary bearers of truth and falsity'. (A sentence is only secondarily, or derivatively, true or false, depending on what proposition it expresses.)
- They are the the 'objects of the attitudes', i.e. they are the things that are known, believed, doubted, etc.

Bolzano's Notion of Proposition (2/2)

- They are nonlinguistic.
- They are nonmental.
- They are not located in space and time.
- Sentences in different languages, or different sentences in the same language, can express the same proposition.
- Two distinct propositions can entail each other.

Kinds of Propositions

A proposition p is called:

- a **necessary truth**, or a **necessity**, iff it is true at every world.
- a **possibility** iff it is true at some world.
- a **truth** iff it is true at the actual world.
- **contingent** iff it is true at some world and false at some world.
- a **falsehood** iff it is false at the actual world.
- a **necessary falsehood**, or an **impossibility**, or a **contradiction**, iff it is true at no world.
- a **fact** of w iff it is true at w.

Formalizing Agnostic Possible Worlds Semantics

- The theory is written in HOL.
- For now, it is a static theory, i.e. no modelling of context.
- Plummer and Pollard, pp. 207-209, show how to extend the agnostic theory to either a tractarian or a kripkean (specifically, montagovian) theory if desired.
- Although frankly, we don't see much linguistic motivation for making such extensions.
- Later we will use our static asgnostic theory as the basis for a *dynamic* theory.

Types

- **Basic types provided by the logic:**

  T (the unit type, used for dummy meanings)

  t (truth values, the type of formulas; also used for extensions of propositions)

  the logic also supplies the type constructors $\wedge$, $\vee$, and $\rightarrow$

- **Nonlogical basic types**

  e (entities)

  p (propositions)

  w (worlds)

Note: We use the following type abbreviations:

a. $\mathrm{p}_{0} =_{\mathrm{def}} \mathrm{p}$

b. $\mathrm{p}_{n+1} =_{\mathrm{def}} \mathrm{e} \rightarrow \mathrm{p}_{n}$

Some Basic Nonlogical Constants

$\vdash @ : \mathrm{p} \rightarrow \mathrm{w} \rightarrow \mathrm{t}$

The is-true-at relation.

$\vdash \mathsf{facts} : \mathrm{w} \rightarrow \mathrm{p} \rightarrow \mathrm{t}$

The function mapping each world to its set of **facts** (the propositions true at that world).

$\vdash$ $\mathsf{entails}$: p $\to$ p $\to$ t

Entailment (written infix).

$\vdash \equiv$ p $\to$ p $\to$ t

Mutual entailment, also called (truth-conditional) equivalence (written infix).

Axioms for the Basic Nonlogical Constants

$\vdash \forall_w.(\mathsf{facts}\ w) = \lambda_p.p@w$

$\vdash \forall_{pq}.(p\ \mathsf{entails}\ q) \leftrightarrow \forall_w.p@w \to q@w$

$\vdash \forall_{pq}.(p \equiv q) \leftrightarrow ((p\ \mathsf{entails}\ q) \wedge (q\ \mathsf{entails}\ p))$

Equivalently, the axiom for equivalence could have been written as:

$\vdash \forall_{pq}.(p \equiv q) \leftrightarrow \forall_w.(p@w) \leftrightarrow (q@w)$

This becomes relevant later when we generalize the notion of equivalence from propositions to all types of meanings.

Some Constants for Word Meanings

$\vdash$ p : e (Pedro)
$\vdash$ c : e (Chiquita)
$\vdash$ m : e (Maria)
$\vdash$ donkey : p_{1}
$\vdash$ farmer : p_{1}
$\vdash$ rain : p
$\vdash$ yell : p_{1}
$\vdash$ kick : p_{2}
$\vdash$ give : p_{3}
$\vdash$ $\mathsf{believe}$: e $\to$ p $\to$ p
$\vdash$ $\mathsf{persuade}$: e $\to$ e $\to$ p $\to$ p
$\vdash$ every : $\mathrm{p}_{\mathit{1}} \to \mathrm{p}_{\mathit{1}} \to$ p
$\vdash$ some : $\mathrm{p}_{\mathit{1}} \to \mathrm{p}_{\mathit{1}} \to$ p

Constants for the Propositional Connectives

$\vdash \mathsf{truth} : \mathrm{p}$ (a necessary truth)

$\vdash \mathsf{falsity} : \mathrm{p}$ (a necessary falsehood)

$\vdash \mathsf{not} : \mathrm{p} \to \mathrm{p}$ (propositional negation)

$\vdash \mathsf{and} : \mathrm{p} \to \mathrm{p} \to \mathrm{p}$ (propositional conjunction, the meaning of the sentence coordinator *and*, written infix)

$\vdash \mathsf{or} : \mathrm{p} \to \mathrm{p} \to \mathrm{p}$ (propositional disjunction, the meaning of the sentence coordinator *or*, written infix)

$\vdash \mathsf{implies} : \mathrm{p} \to \mathrm{p} \to \mathrm{p}$ (propositional implication, the meaning of the subordinator *if*)

Axioms for the Propositional Connectives (1/2)

$\vdash \forall_w.\mathsf{truth}@w$

$\vdash \forall_w.\neg(\mathsf{falsity}@w)$

$\vdash \forall_{pw}.(\mathsf{not}\ p)@w \leftrightarrow \neg(p@w)$

$\vdash \forall_{pqw}.(p\ \mathsf{and}\ q)@w \leftrightarrow (p@w \wedge q@w)$

$\vdash \forall_{pqw}.(p\ \mathsf{or}\ q)@w \leftrightarrow (p@w \vee q@w)$

$\vdash \forall_{pqw}.(p\ \mathsf{implies}\ q) \leftrightarrow (p@w \to q@w)$

Axioms for the Propositional Connectives (2/2)

- These axioms say that, in any interpretation, the set of propositions form a **preboolean algebra**, with entailment as the preorder. (A preboolean algebra is like a boolean algebra, except that all the expected equalities are replaced by equivalences.)
- For each world, the set of facts for that world form a **maximal consistent set** (an **ultrafilter** over over the algebra of propositions), i.e.:
  - it is closed under entailment
  - it is closed under conjunction
  - for each proposition, it has either that proposition or its negation as a member, but not both.
- See Plummer and Pollard pp. 206-207 for the proof.

Propositional Quantifiers

These will be used to analyze the meanings of determiners such as *every, all, some, a(n)*, and *no*.

Constants:

$\vdash \mathsf{forall}\ : (\mathrm{e} \to \mathrm{p}) \to \mathrm{p}$

$\vdash \mathsf{exists}\ : (\mathrm{e} \to \mathrm{p}) \to \mathrm{p}$

Axioms:

$\vdash \forall_{Pw}.(\mathsf{forall}\ P)@w \leftrightarrow \forall_x.(P\ x)@w$

$\vdash \forall_{Pw}.(\mathsf{exists}\ P)@w \leftrightarrow \exists_x.(P\ x)@w$

Some Useful Abbreviations

$\mathsf{that} =_{\mathrm{def}} \lambda_{PQx}.(P\ x)\ \mathsf{and}\ (Q\ x)$ (property conjunction, meaning of the relativizer *that*)

$\mathsf{some} =_{\mathrm{def}} \lambda_{PQ}.\mathsf{exists}(\lambda_x.(P\ x)\ \mathsf{and}\ (Q\ x)) = \lambda_{PQ}.\mathsf{exists}(P\ \mathsf{that}\ Q)$ (meaning of the determiner *some*)

$\mathsf{every} =_{\mathrm{def}} \lambda_{PQ}.\mathsf{forall}(\lambda_x.(P\ x)\ \mathsf{implies}\ (Q\ x))$ (meaning of the determiner *every*)

Meaning Types

- Not all types of the semantic theory are types of meanings.
- For example, there are no meanings of type t or of type w.
- We recursively define the set of **meaning types** as follows:
  - T, e and p are **basic** meaning types.
  - If A and B are meaning types, then:

    $A \wedge B$ is a **product** meaning type.

    $A \to B$ is a **functional** meaning type.
  - Nothing else is a meaning type.

Extension Types

- For each meaning type A, there is a corresponding type $\mathrm{Ext}(A)$ for the extensions of meanings of type A.
- Ext is recursively defined as follows:
  - $\mathrm{Ext}(\mathrm{T}) = \mathrm{T}$
  - $\mathrm{Ext}(\mathrm{e}) = \mathrm{e}$
  - $\mathrm{Ext}(\mathrm{p}) = \mathrm{t}$
  - $\mathrm{Ext}(A \wedge B) = \mathrm{Ext}(A) \wedge \mathrm{Ext}(B)$
  - $\mathrm{Ext}(A \to B) = A \to \mathrm{Ext}(B)$
    (*not* $\mathrm{Ext}(A) \to \mathrm{Ext}(B)$)

Extension of a Meaning at a World

- We introduce a family of constants (written infix)

  $$\vdash @_A \ : A \to \mathrm{w} \to \mathrm{Ext}(A)$$

  where A ranges over meaning types.
- $a@_A w$ is read 'the extension of a at w'.
- Axioms:

  $\vdash \forall_{uw}.u@_{\mathrm{T}}w = u$ ($A = \mathrm{T}$)

  $\vdash \forall_{xw}.x@_{\mathrm{e}}w = x$ ($A = \mathrm{e}$)

  $\vdash \forall_{pw}.p@_p w = p@w$ ($A = \mathrm{p}$)

  $\vdash \forall_{zw}.z@_A w = (\pi(z)@w, \pi'(z)@w)$ (A a product type)

  $\vdash \forall_{fw}.f@w = \lambda_x.(f\ x)@w$ (A a functional type)

  Note: Because of the axiom for $A = \mathrm{p}$, henceforth the subscript on a_A can be omitted.

Equivalence of Meanings, Generalized

- Recall that two propositions are **equivalent** iff they are true at the same worlds, i.e. $p \equiv q$ iff for every world w, $p@w = q@w$.
- More generally, we can now say two meanings a and b of the same type are **equivalent** iff, for every world w, a and b have the same extension at w.

- That is, for each meaning type A, we can define **meaning equivalence** by

$$\equiv_A =_{\text{def}} \lambda_{xy}.\forall_w.(x@w = y@w)$$

Note that for $A = \text{p}$, this coincides with the original definition of (truth-conditional) equivalence.

Extensional Equality

- Two meanings a and b are called **extensionally equal** at w iff they have the same extension at w.
- We express extensional equality using he family of constants (written infix)

$$\vdash \mathsf{exteq}_A \ : A \to A \to p$$

- Axiom:

$$\vdash \forall_{xyw}.(x \ \mathsf{exteq} \ y)@w \leftrightarrow (x@w = y@w)$$

Hyperintensional Equality

- Two meanings a and b are called **hyperintensionally equal** at w iff they are the same meaning.

  So if a and b are hyperintensionally equal at some world w, then they are hyperintensionally equal at *all* worlds.
- We express hyperintensional equality using the family of constants (written infix)

$$\vdash \mathsf{equals}_A \ : A \to A \to p$$

- Axiom:

$$\vdash \forall_{xyw}.(x \ \mathsf{equals} \ y)@w \leftrightarrow (x = y)$$

Montague Semantics (1/2)

Montague semantics is the theory sketched above with the following additions:

- ‘p is now taken to abbbreviat w $\to$ t (so that p is no longer basic)
- the **montagovian** axiom

$$\vdash @ = \lambda_p.p$$

  That is: for p to be true at w is for w to be a set-theoretic member of p.
- This is a *kripkean* theory, in the sense that w is a basic type (i.e. worlds are theoretical primitives, not constructed from other things).

Montague Semantics (2/2)

Advantages:

- simplicity: propositions form a powerset algebra.
- familiarity: most linguistic semanticists have a working familiarity with it.

Disadvantages:

- Insufficiently fine-grained meanings: equivalent meanings must be identical.
- As long as there are infinitely many worlds (or equivalently, infinitely many propositions), necessarily not every maximal consistent set of propositions has a world for which it is the set of facts (only those which are *principal* ultrafilters). Then it must be explained *which* ones 'count' as worlds.

Tractarian Semantics (1/2)

- **Tractarian semantics** is the theory sketched above with the addition of two axioms which jointly say that the function facts is a bijection onto the set of maximal consistent sets of propositions.
- The first axiom says that facts is injective.
- The second axiom says that every maximal consistent set is the set of facts for some world.
- The precise formulation of these axioms can be found in Plummer and Pollard pp. 208-209,

Note: Tractarian semantics so defined is a little more general than Wittgenstein's original version, where worlds *are* maximal consistent sets, rather than merely being in one-to-one correspondence with them. But Wittgenstein's version is hard to state in standard HOL.

Tractarian Semantics (2/2)

Advantages:

- Fine-grained meanings (there can be distinct but equivalent meanings)
- No need to explain why nonprincipal ultrafilters don't correspond to worlds: they do.

Disadvantages:

- involves slightly more algebra (preboolean algebras and maximal consistent sets)
- Linguists aren't as familiar with the relevant philosophers (e.g. Wittgenstein, C.I. Lewis, Adams, Plantinga, Lycan) as they are with the ones who advocated Montague semantics (e.g. Montague, D. Lewis, Stalnaker, Kaplan)