Problem Set One

Carl Pollard
The Ohio State University
Advances in Logical Grammar

June 12, 2012

These problems are due in class, or by 17:00 by email, on Monday June 18, 2012.

Problem 1

Give a LL natural-deduction proof tree for the following sequent, known as **Generalized Contraposition**:

$$A \multimap B \vdash (B \multimap C) \multimap A \multimap C$$

Note: Traditionally, the term **Contraposition** is used for the special case with $B = \mathrm{F}$ of the counterpart of this sequent in intuitionistic or classical PL (with $\multimap$ replaced by $\rightarrow$).

Problem 2

Give a LL natural-deduction proof tree for the following sequent, known as **Geach's Law**:

$$A \multimap B \vdash (C \multimap A) \multimap C \multimap B$$

Problem 3

In natural deduction, we say that an inference rule is **derivable** if we *could have* proved the conclusion if the premiss(es) had been provable. In other words, we derive an inference rule by presenting a proof tree whose conclusion is that of the rule in question, and in which we allow the premisses of that rule, in addition to the usual axioms, as leaves of the proof tree. For example, the following inference rule, here called CHP (Converse of Hypothetical Proof):

$$\frac{\Gamma \vdash A \multimap B}{\Gamma, A \vdash B}\ \text{CHP}$$

is LL-derivable as follows:

$$\frac{\Gamma \vdash A \multimap B \qquad A \vdash A}{\Gamma, A \vdash B}\ \text{MP}$$

Once derived, a rule can be used in any proof just as if it were one of the original rules of the proof system.

Now, show that the following rule, known as **Curry's Law**, is PIPL-derivable:

$$\frac{\Gamma \vdash (A \wedge B) \to C}{\Gamma \vdash A \to B \to C}\ \text{Curry}$$

Note: The term-labelled counterpart of Curry in TLC is what underlies the widespread practice of "currying" functions.

Problem 4

Show that in LL, the sequent $\vdash A \multimap B$ is derivable iff the following inference rule schema (call it R) is:

$$\frac{\Gamma \vdash A}{\Gamma \vdash B}\ \text{R}$$

Note: Later we will use this fact to show that adding an 'inaudible' lexical entry to grammar is logically equvalent to adding a unary (i.e. nonbranching) grammar rule.

Problem 5

Derive the following inference rule schema, called **Composition**:

$$\frac{\Gamma \vdash B \multimap C \qquad \Delta \vdash A \multimap B}{\Gamma, \Delta \vdash A \multimap C}\ \text{Comp}$$

Note: Composition is used to shorten syntactic analyses of unbounded dependencies (by eliminating a trace and the corresponding hypothetical proof step).