FSLT Semantics Exercise

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1. Formalise the following sentences in propositional logic! (Translate basic sentences like "it rains" or "Steve comes home late" to propositional constants p, q, r) .

a. When it rains, it pours.

The sentence has 2 meanings: temporal and inferential.

p= "it rains"

q= "it pours"

Inferential meaning (equivalent to "If it rains, it pours"):

$p \to q$

Temporal meaning (equivalent to "During the time it rains, it pours"):

Let's introduce a time variable t, that will keep track of the time state of the world (however, time is not a conventional meaning of this variable, i.e. it is impossible to imply this meaning from the formula only, and a comment is always needed):

$\forall t(p\&t) \to (q\&t)$

b. Sam wants a dog, but Alice prefers cats.

Propositional logic doesn't provide tools for expressing contradiction, but I can introduce a constant d which appearance in the formula would signify that there is no world in which both of the propositions that are arguments of verbs "wants" and "prefers" can be true. Again, the meaning of this constant is not conventional. Moreover, the way the value of this constant is formulated shows that it impossible to express the meaning of "but" in terms of propositional logic, because one would need to decompose p and q into main predicates $want$ and $prefer$ and propositions $h1$ = "have a dog" and $h2$ = "have a cat".

p= "Sam wants a dog"

q= "Alice prefers cats"

$p\&q\&d$

c. I will make the dishes if you cook.

p= "I will make the dishes"

q= "you cook"

$q \to p$

d. I will make the dishes only if you cook

p= "I will make the dishes"

q= "you cook"

$(q \to p)\&(\neg q \to \neg p)$

In other terms,

$p \leftrightarrow q$

e. Marsha won't go out with John unless he shaves off his beard and stops drinking.

p= "Marsha will go out with John"

q= "he shaves off his beard"

$r=$ "he stops drinking"

$(\neg q \& \neg r \rightarrow \neg p) \& (q \& r \rightarrow p)$

f. The stock market advances when public confidence in the economy is rising.

Same reasoning as in 1 applies here. The sentence again has an inferential and a temporal meanings. I will introduce the same time variable t with the same properties:

$p=$ "the stock market advances"

$q=$ "public confidence in the economy is rising"

Inferential meaning (equivalent to "If public confidence in the economy is rising, the stock market advances"):

$q \rightarrow p$

Temporal meaning (equivalent to "During the times when public confidence in the economy is rising, the stock market advances):

$\forall t(q \& t) \rightarrow (p \& t)$

g. John and Bill are going to the movies, but not Tom.

$p=$ "John and Bill are going to the movies"

$q=$ "Tom is going to the movies"

Propositional logic doesn't have means for representing discourse contrast enclosed in "but". Logical representation of the propositional meaning of the sentence is:

$p \& \neg q$

Contrast doesn't belong to the propositional level of the meaning. Is it what is called pragmatics?

h. If Mary hasn't got lost or had an accident, she will be here in 5 minutes.

$p=$ "Mary has got lost"

$q=$ "Mary has had an accident"

$r=$ "Mary will be here in 5 minutes"

$(\neg p \& \neg q) \rightarrow r$

I don't know of special means of propositional logic for representation of temporal and aspectual sematics.

One general note:

Striktly speaking, $q \rightarrow p$ is not equivalent to a conditional sentence in natural language. For cases $q = 1$, the formula represents the natural language sentence correctly. But for the case when the condition is false, the correct value of the sentence would be "I don't know", but the value of the formula is "true".

2. Check with the truth-table method, whether the following formulae are logically valid, contradictory, or contingent (i.e. neither valid nor contradictory)! .

a. $((p \vee \neg q) \wedge q)$

p	q	¬q	p∨¬q	(p∨¬q)∧q
1	1	0	1	1
1	0	1	1	0
0	1	0	0	0
0	0	1	1	0

This statement is false when q is true, in other cases it is true, i.e. it is contingent.

b. $((p \wedge q) \rightarrow (p \vee r))$

p	q	p∧q	r	p∨r	(p∧q)→(p∨r)
1	1	1	1	1	1
1	1	1	0	1	1
1	0	0	1	1	1
1	0	0	0	1	1
0	1	0	1	1	1
0	1	0	0	0	1
0	0	0	1	1	1
0	0	0	0	0	1

This statement is always true, i.e. it is valid.

c. $(\neg p \wedge \neg(p \rightarrow q))$

p	¬p	q	p→q	¬(p→q)	¬p∧¬(p→q)
1	0	1	1	0	0
1	0	0	0	1	0
0	1	1	1	0	0
0	1	0	1	0	0

This statement is always false, i.e. contradictory.

3. Check with the truth-table method whether entailment holds in the following cases: .

a. $(p \rightarrow \neg q), (r \rightarrow q), (\neg r \rightarrow q) \models \neg p$?

	p	q	¬q	p → ¬q	r	r → q	¬r	¬r → q	¬p
1	1	1	0	0	1	1	0	1	0
2	1	1	0	0	0	1	1	1	0
3	1	0	1	1	1	0	0	1	0
4	1	0	1	1	0	1	1	0	0
5	0	1	0	1	1	1	0	1	1
6	0	1	0	1	0	1	1	1	1
7	0	0	1	1	1	0	0	1	1
8	0	0	1	1	0	1	1	0	1

All the formulae in the left part are true in cases 5 and 6. $\neg p$ is true in these cases as well. The entailment holds.

b. $(q \vee r), ((q \wedge r) \rightarrow s) \models (q \rightarrow s)$?

	q	r	q $\vee$ r	q $\wedge$ r	s	((q$\wedge$ r) $\to$ s)	(q $\to$ s)
1	1	1	1	1	1	1	1
2	1	1	1	1	0	0	0
3	1	0	1	0	1	1	1
4	1	0	1	0	0	1	0
5	0	1	1	0	1	1	1
6	0	1	1	0	0	1	1
7	0	0	0	0	1	1	1
8	0	0	0	0	0	1	1

All the formulae in the left part are true in cases 1, 3, 4, 5, 6. However, the formula in the right part is false in the forth case. The entailment doesn't hold.

4. Translate the following sentences to FOL. .

a. John admires someone.

$\exists x(admire(John, x))$

b. John admires himself.

$admire(John, John)$

c. Bill and Mary help each other.

$help(Bill, Mary)\&help(Mary, Bill)$

d. A student reads an interesting book

$\exists x\exists y(student(x)\&interesting(y)\&book(y)\&read(x, y))$

e. Peter reads only interesting books.

$\forall x(read(Peter, x) \to (interesting(x)\&book(x)))$

f. No one is loved by everyone.

$\forall y\exists x(\neg love(y, x))$

g. All but one student passed (the exam).

$\exists x(\neg pass(x)\&student(x)\&\forall y(x \neq y \to pass(y)))$

h. Only Peter flunked.

$\exists x(flunk(x)\&x = Peter\&\forall y(x \neq y \to \neg flunk(y)))$

i. Exactly one student flunked.

$\exists!x(flunk(x))$

5. Are the following formulae logically valid, contradictory (false in all model structures), or contingent (neither valid nor contradictory)? .

a. $\exists x(F(x) \wedge \neg F(x))$

This formula is contradictory. The intersection of a formula and its negation can never be true, because the intersection is only valid when both of its members are valid, and as it follows from the definition of the negation operator, there is no such case when a formula and its negation are both valid.

b. $(\exists xF(x) \vee \exists x\neg F(x))$

This formula is always true, i.e. valid.

Suppose it is not the case. Then, there should be a model where it is not true. I will try to construct it. For the formula to be false, it is necessary that $\exists xF(x) = 0$ and $\exists x\neg F(x) = 0$. But if $\exists xF(x) = 0$, then$\neg(\exists xF(x)) = 1$, then

$\exists x \neg F(x) = 1$ and vice versa. There is no such model in which they both are false. It means that the formula is always true, i.e. valid.

c. $(\forall x F(x) \vee \forall x \neg F(x))$

This formula is contigent. To prove this, I need to construct a model where it is true and a model where it is false.

To construct the first model, I only need to accept the condition that $\forall x F(x)$ is not true. To construct the second model, I need to accept that $\forall x F(x)$ is not ture and$\forall x \neg F(x)$ is not true in the same model. It means that for some x $F(x)$ should be true and for some other x $F(x)$ should be false, i.e. I should find a function that gives different results for different x values. Say, $F = nasty$. I will take the model, where $nasty(John)$ is true and $nasty(Mary)$ is false. Then, for one $x = John$ $F(x)$ will be true and for $x = Mary$ $F(x)$ will be false.

Check whether the entailment holdsin the following cases (through semantic interpretation of the involved formulas): .

a. $\forall x F(x), G(a) \models \exists x(F(x) \wedge G(x))$

Say we have

a) the model M={D,I}, where

D is the domain of constants

I is the interpretation function

b) variable assignment function g.

Truth conditions for formulae in the left part:

$[\forall x F(x)]^{M,g} = 1$ iff

for each constant d in the domain of constants D $[F(x)]^{M,g} = 1$ iff

for each $d \in D$ $[x]^{M,g[d/x]} \in [F]^{M,g[d/x]}$ iff

for each $d \in D$ $d \in I(F)$.

$[G(a)]^{M,g} = 1$ iff

$I(a) \in I(G)$.

Truth conditions for the right part:

$\exists x(F(x) \wedge G(x)) = 1$ iff

there exists at least one $d \in D$ such that $[F(x) \wedge G(x)]^{M,g} = 1$ iff

there exists at least one $d \in D$ such that

$[F(x)]^{M,g} = 1$ and $[G(x)]^{M,g} = 1$ iff

there exists at least one $d \in D$ such that

$[x]^{M,g[d/x]} \in [F]^{M,g[d/x]}$ and $[x]^{M,g[d/x]} \in [G]^{M,g[d/x]}$ iff

there exists at least one $d \in D$ such that

$d \in I(F)$ and $d \in I(G)$.

Both formulae from the right part are true when the first is true and the second is true. If for each $d \in D$ $d \in I(F)$, then $a \in I(F)$. Then, conjunction of the truth conditions for both formulae is $a \in I(F)$ and$a \in I(G)$. Since a is a constant from the domain D, the truth conditions for the right part are satisfied. That is, the entailment betweeen the left and the right parts holds.

b. $F(a), \exists x(F(x) \wedge G(x)) \models G(a)$

We have

a) the model M={D,I}, where

D is the domain of constants

I is the interpretation function

b) variable assignment function g.

Truth conditions for formulae in the left part:

$\exists x(F(x) \wedge G(x)) = 1$ iff there exists at least one $d \in D$ such that $d \in I(F)$ and $d \in I(G)$.

$[F(a)]^{M,g} = 1$ iff $I(a) \in I(F)$.

The truth conditions for the right part:

$[G(a)]^{M,g} = 1$ iff $I(a) \in I(G)$.

Suppose we choose the model such that there are constants a and b such that $I(b) \neq I(a)$ and $I(a) \in I(F)$ and $I(b) \in I(F)$ and $I(b) \in I(G)$. In this model, $I(a) \notin I(G)$, but the truth conditions for both formulae in the left part are true. For such model, the entailment doesn't hold, i.e. it doesn't hold in the general case.

c. $\forall x(F(x) \leftrightarrow \neg G(x)), F(a), G(b) \models \neg a = b$

Truth conditions for formulae in the left part:

$\forall x(F(x) \leftrightarrow \neg G(x)) = 1$iff

for each $d \in D$ $[F(x) \leftrightarrow \neg G(x)]^{M,g} = 1$ iff

for each $d \in D$ $[F(x)]^{M,g} = [\neg G(x)]^{M,g}$in 2 cases only:

1. for each $d \in D$ $[F(x)]^{M,g} = 1$ and $[\neg G(x)]^{M,g} = 1$

or

2. for each $d \in D$ $[F(x)]^{M,g} = 0$ and $[\neg G(x)]^{M,g} = 0$

iff

1. for each $d \in D$ $d \in I(F)$ and $d \notin I(G)$

or

2. for each $d \in D$ $d \notin I(F)$ and $d \in I(G)$

$F(a)$iff $I(a) \in I(F)$

$G(b)$iff $I(b) \in I(G)$

Truth conditions for the formula in the right part:

$\neg a = b$iff $I(a) \neq I(b)$.

Let's check wha are the truth conditions for the conjucntion of the three formulae in the left part.

In D there should be at least 2 constants a and b. I am now inserting them into the first formula and check what will follow.

For a:

$[F(x)]^{M,g[a/x]} = 1$ and $[\neg G(x)]^{M,g[a/x]} = 1$ iff $a \in I(F)$and $a \notin I(G)$

or

$[F(x)]^{M,g[a/x]} = 0$ and $[\neg G(x)]^{M,g[a/x]} = 0$ iff $a \notin I(F)$ and$a \in I(G)$

For b:

$[F(x)]^{M,g[b/x]} = 1$ and $[\neg G(x)]^{M,g[b/x]} = 1$ iff $b \in I(F)$and $b \notin I(G)$

or

$[F(x)]^{M,g[b/x]} = 0$ and $[\neg G(x)]^{M,g[b/x]} = 0$ iff $b \notin I(F)$ and$b \in I(G)$

The conjucntion with the truth conditions for two other formulae yield the following set of conditions:

$a \in I(F)$and $a \notin I(G)$ and $b \notin I(F)$ and$b \in I(G)$.

Since I is a function, it should always give the same result for the same argument given the same model M and the same variable assignment function g, i.e. if $(I(\phi) = I(\varphi))$, then $(\phi = \varphi)$. Then, if $(I(\phi) \neq I(\varphi))$, then $(\phi \neq \varphi)$. From this, it follows that if $a \in I(F)$ and $b \notin I(F)$, then $I(F(a)) \neq I(F(b))$. Since $I(F)$ is the same, $I(a) \neq I(b)$, that is the truth conditions of the right part. The entailment holds.