Foundations of Language Science and Technology

Semantics 1

Manfred Pinkal
Saarland University

Semantic Processing

Meaning Representation

Interpretation

Text/ Utterance

Laura is sleeping

Semantic Processing: Interpretation + Inference

Inference

Inference in Text Understanding

Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf at a time. At birth a bottlenose dolphin calf is about 90-130 cms long and will grow to approx. 4 metres, living up to 40 years. They are highly sociable animals, living in pods which are fairly fluid, with dolphins from other pods interacting with each other from time to time.

- Are dolphins mammals?
- Are dolphins vertebrates?
- Are dolphins birds?
- Is Flipper a dolphin?

Inference in Language Technology

- Question Answering:
  Who was Lincoln's Secretary of State?
  William H. Seward served as Secretary of State under President Abraham Lincoln.
- Document Retrieval / information extraction:
  - *Airbus sells five A380 planes to China Southern for 220 million Euro*
  - *China Southern buys five A380 planes from Airbus for 220 million Euro*
  - *Five A380 planes will go for 220 million Euro to China Southern*
- Summarisation: Can the summary be inferred from the full text?
- Machine Translation, Dialogue, ...

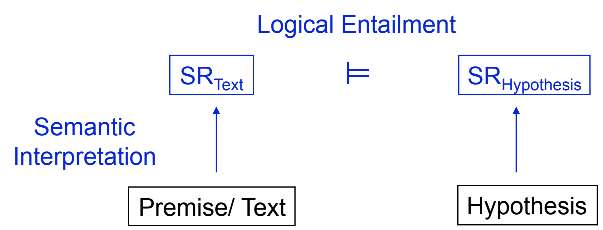
The Story

- Modelling natural-language inference as deduction in a framework of truth-conditionally interpreted logic appears intuitive and straightforward.
- But: Logical methods are expensive and lack robustness and coverage.
- Corpus-based statistical methods for modelling inference are inexpensive and have no coverage problem.
- But: Shallow statistical models of inference are inherently imprecise and resist a satisfactory intuitive interpretation.
- But: There are highly promising approaches, which combine deep logic-based and shallow statistical methods.
- We will look at Bill MacCartney's doctoral dissertation on "Natural-language Inference" as one of the most interesting approaches.
  (Stanford University, June 2009)

The Story

- Modelling natural-language inference as deduction in a framework of truth-conditionally interpreted logic appears intuitive and straightforward.

Logical Entailment



Are dolphins mammals?

Logical Entailment: ✓

$\forall d\,(dolphin(d) \rightarrow mammal(d) \land \neg fish(d))$ $\models$ $\forall d\,(dolphin(d) \rightarrow mammal(d))$

Semantic Interpretation

Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf at a time. At birth a bottlenose dolphin calf ...

Dolphins are mammals.

Is Flipper a mammal?

$\forall d\,(dolphin(d) \rightarrow mammal(d) \land \neg fish(d))$

$dolphin(flipper) \rightarrow mammal(flipper) \land \neg fish(flipper)$ $dolphin(flipper)$

$mammal(flipper) \land \neg fish(flipper)$

$mammal(flipper)$

Logical Entailment: ✓

$\forall d\,(dolphin(d) \rightarrow mammal(d) \land \neg fish(d))$ $\models$ $mammal(flipper)$

Semantic Interpretation

Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf at a time. At birth a bottlenose dolphin calf ...

Flipper is a mammal.

Are dolphins fish?

Logical Entailment: ✘

$\forall d\,(dolphin(d) \rightarrow mammal(d) \land \neg fish(d))$ $\models$ $bird(flipper)$

Semantic Interpretation

Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf at a time. At birth a bottlenose dolphin calf ...

Flipper is a bird.

Recommended Reading

- Textbook: L.T.F. Gamut, Logic, Language, and Meaning. University of Chicago Press 1991

  Volume1: Introduction to Logic.

  Volume2: Intensional Logic and Logical Grammar.

Predicate Logic – Vocabulary

- The vocabulary of the language of predicate logic:
  - Logical symbols
    - Connectives: ¬, ∧, ∨, →, ↔
    - Quantifiers: ∀, ∃
    - Equality: =
  - Infinite set of individual variables:
    - VAR = { x, y, z, … }
  - Arbitrary set of individual constants:
    - CON = { a, b, c, … }
  - For every $n \geq 0$, an arbitrary, possibly empty set of n-ary predicate symbols: $PRED^n$ = { P, Q, … }

Predicate Logic – Syntax

- Terms: TERM = CON ∪ VAR
- (Well-formed) Formulae: the smallest set such that
  (1) If R is an n-ary predicate symbol, and $t_1, …, t_n$ are terms, then $R(t_1, …, t_n)$ is a wff.
  (2) If t_1, t_2 are terms, then $t_1 = t_2$ is a wff.
  (3) if φ, ψ are wff, then ¬φ, (φ ∧ ψ), (φ ∨ ψ), (φ → ψ), and (φ ↔ ψ) are wff.
  (4) if φ is a wff, and x an individual variable, then ∀xφ and ∃xφ are wff.

Predicate Logic – Atomic formulae

(1) If R is an n-ary predicate symbol, and $t_1, …, t_n$ are terms, then $R(t_1, …, t_n)$ is a wff.
(2) If t_1, t_2 are terms, then $t_1 = t_2$ is a wff.

Examples:

– *Flipper is a dolphin*	dolphin(flipper)
– *Bill works*	work(bill)
– *Mary likes John*	like(john, mary)
– *John is taller than Bill*	taller_than(john, bill)
– *John introduces Bill to Mary*	introduce(john, bill, mary)
– *Angela Merkel is the chancellor*	angela_merkel = the_chancellor

Predicate Logic – Complex Formulae

(3) if φ, ψ are wff, then ¬φ, (φ ∧ ψ), (φ ∨ ψ), (φ → ψ), and (φ ↔ ψ) are wff.

Name	Connective	NL Paraphrase
negation	¬p	it is not the case that p
conjunction	(p ∧ q)	p and q
disjunction	(p ∨ q)	p or q
implication	(p → q)	if p then q
equivalence	(p ↔ q)	p if and only if q

Flipper is not a fish. ¬fish(flipper)
If Flipper is a dolphin, he is a mammal.
dolphin(flipper) →mammal(flipper)

Predicate Logic – Complex Formulae

(4) if φ is a wff, and x an individual variable, then ∀xφ and ∃xφ are wff.

Bill reads an interesting book.
∃b (book(b) ∧ interesting(b) ∧ read (bill,b))

Dolphins are mammals, not fish.
∀d (dolphin(d)→mammal(d) ∧¬fish(d))

Dolphins live in pods.
∀d (dolphin(d)→ ∃x (pod(p) ∧ live-in (d,p))

Dolphins give birth to one baby at a time.
∀d (dolphin(d)→ ∀x ∀y ∀t (give-birth-to (d,x,t) ∧ give-birth-to (d,y,t) → x=y)

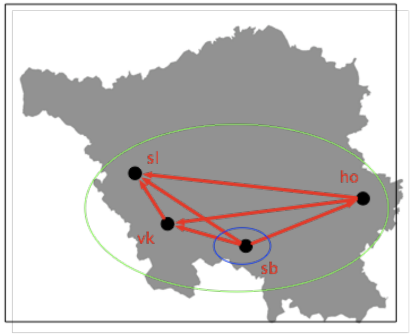
Semantic Interpretation of FOL

- FOL expressions are interpreted with respect to certain situations or states of the world.
- FOL expressions of certain types (terms, relation symbols, formulae) are assigned specific kinds of objects (denotations) by an interpretation function.
- In particular, formulae denote truth values.
- Situations or states of the world (more precisely: the relevant properties of situations and states of the world) are formally represented by model structures.

A Model of Saarland Towns

$M = (U_M, V_M)$

U_M = { sl, vk, ho, sb }



V_M defined by:

- V_M (saarbrücken) = sb
- V_M (völklingen) = vk
- V_M (saarlouis) = sl
- V_M (homburg) = ho
- V_M(larger_than) = { (sb, sl), (sb, vk), (sb, ho), (vk, sl), … }
- V_M(town) = { sl, vk, ho, sb }
- V_M(capital) = { sb }

Model Structures

- A model structure is a pair $M = (U_M, V_M)$, where
  - U_M is a non-empty set (the "model universe"), and
  - V_M is an value assignment function for basic expressions, which assigns
    n-ary relations (over U_M) to n-ary predicate symbols, and elements of U_M to predicate constants:

    $V_M(P) \subseteq U_M^n$, if P is an n-ary predicate symbol

    $V_M(c) \in U_M$, if c is a constant

Interpretation of Atomic Formulae

- Terms: TERM = CON ∪ VAR
- (Well-formed) Formulae: the smallest set such that
  (1) If R is an n-ary predicate symbol, and $t_1, \ldots, t_n$ are terms, then $R(t_1, \ldots, t_n)$ is a wff.
  (2) If t_1, t_2 are terms, then $t_1 = t_2$ is a wff.
  (3) if φ, ψ are wff, then ¬φ, (φ ∧ ψ), (φ ∨ ψ), (φ → ψ), and (φ ↔ ψ) are wff.
  (4) if φ is a wff, and x an individual variable, then ∀xφ and ∃xφ are wff.

Interpretation of Atomic Formulae

- An interpretation function $[[\ ]]^{M,g}$ recursively assigns semantic values $[[\alpha]]^{M,g}$ to all expressions α with respect to a model structure and a variable assignment g.
- Interpretation of terms:
  $[[c]]^{M,g} = V_M(c)$ for all individual constants c
  $[[x]]^{M,g} = g(x)$
- Interpretation of atomic expressions:
  $[[R(t_1, \ldots, t_n)]]^{M,g} = 1$ iff $([[t_1]]^{M,g}, \ldots, [[t_n]]^{M,g}) \in V_M(R)$
  $[[t_1 = t_2]]^{M,g} = 1$ iff $[[t_1]]^{M,g} = [[t_2]]^{M,g}$

  Example:
  larger_than(saarbrücken, homburg) =1
  iff ([[saarbrücken]]^{M,g}, [[homburg]]^{M,g} $\in$ V_M(larger_than)
  iff (V_M(saarbrücken), V_M(homburg)) $\in$ V_M(larger_than)
  iff (sb, ho) $\in$ V_M(larger_than)

Predicate Logic – Syntax

- Terms: TERM = CON ∪ VAR
- (Well-formed) Formulae: the smallest set such that
  (1) If R is an n-ary predicate symbol, and $t_1, \ldots, t_n$ are terms, then $R(t_1, \ldots, t_n)$ is a wff.
  (2) If t_1, t_2 are terms, then $t_1 = t_2$ is a wff.
  (3) if φ, ψ are wff, then ¬φ, (φ ∧ ψ), (φ ∨ ψ), (φ → ψ), and (φ ↔ ψ) are wff.
  (4) if φ is a wff, and x an individual variable, then ∀xφ and ∃xφ are wff.

Interpretation of connectives

$[[\neg\varphi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 0$
$[[\varphi \wedge \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 1$ and $[[\psi]]^{M,g} = 1$
$[[\varphi \vee \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 1$ or $[[\psi]]^{M,g} = 1$
$[[\varphi \rightarrow \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 0$ or $[[\psi]]^{M,g} = 1$
$[[\varphi \leftrightarrow \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = [[\psi]]^{M,g}$

- Connectives in predicate logic are truth-functional: Their truth-value is completely determined by the truth-values of their constituent clauses.
- The interpretation of connectives can be represented by truth-tables.

Truth Tables for Connectives

A	¬A
0	1
1	0

A	B	(A∧B)
0	0	0
0	1	0
1	0	0
1	1	1

A	B	(A∨B)
0	0	0
0	1	1
1	0	1
1	1	1

A	B	(A→B)
0	0	1
0	1	1
1	0	0
1	1	1

A	B	(A↔B)
0	0	1
0	1	0
1	0	0
1	1	1

Composite Truth Tables

	A	B	¬A	¬B	¬A ∧ ¬B	¬(¬A ∧ ¬B)
M_1	1	1	0	0	0	1
M_2	1	0	0	1	0	1
M_3	0	1	1	0	0	1
M_4	0	0	1	1	1	0

Predicate Logic – Syntax

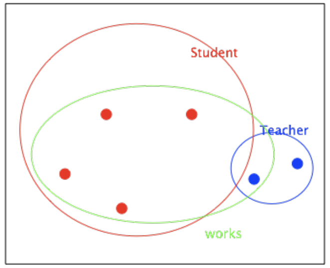
- Terms: TERM = CON ∪ VAR
- (Well-formed) Formulae: the smallest set such that
  (1) If R is an n-ary predicate symbol, and $t_1, \ldots, t_n$ are terms, then $R(t_1, \ldots, t_n)$ is a wff.
  (2) If t_1, t_2 are terms, then $t_1 = t_2$ is a wff.
  (3) if φ, ψ are wff, then ¬φ, (φ ∧ ψ), (φ ∨ ψ), (φ → ψ), and (φ ↔ ψ) are wff.
  (4) if φ is a wff, and x an individual variable, then ∀xφ and ∃xφ are wff.

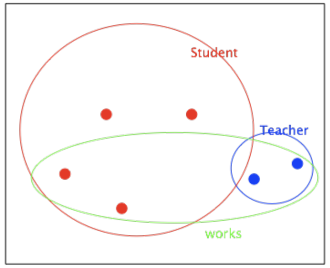
Quantifier Interpretation– Preliminary!

- A preliminary formulation of a general interpretation function for quantified formulae:
  $[[\exists xA]]^{M,g} = 1$ iff
  there is at least one variable assignment g' such that $[[A]]^{M,g'} = 1$
  $[[\forall xA]]^{M,g} = 1$ iff
  $[[A]]^{M,g'} = 1$ for all variable assignments g'.

An Example

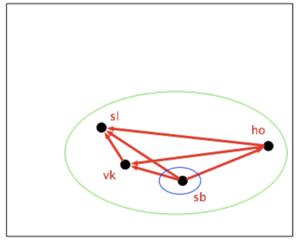
- "Every student works" $\Rightarrow \forall x(\text{student}(x) \rightarrow \text{work}(x))$
- True in model M1, false in model M2.
- M1 M2





Another example

- $[[\ \exists x(\text{town}(x) \wedge \text{larger\_than}(x, \text{völklingen}))\ ]]^{M,g} = 1$
  iff there is g' such that
  $[[\ \text{town}(x) \wedge \text{larger\_than}(x, \text{völklingen}))\ ]]^{M,g'} = 1$
  $\Leftrightarrow [[\ \text{town}(x)]]^{M,g'} = 1 \wedge [[\ \text{larger\_than}(x, \text{völklingen}))\ ]]^{M,g'} = 1$
  $\Leftrightarrow [[x]]^{M,g'} \in V_M(\text{town})$ and
  $([[x]]^{M,g'}, [[\text{völklingen}]]^{M,g'}) \in V_M(\text{larger\_than})$
  $\Leftrightarrow g'(x) \in V_M(\text{town})$ and
  $(g'(x), V_M(\text{völklingen})) \in V_M(\text{larger\_than})$
- $\exists x(\text{town}(x) \wedge \text{larger\_than}(x, \text{völklingen}))$
  is true in the Saarland model: Saarbrücken and Homburg are verifying instantiations for x



Variable Assignments

- Attention: The interpretion function for quantifiers is incorrect for the general case of formulae containing several nested quantifiers. We need a notion of a modified variable assignment function. We do not have the time to treat it in the course. Definitions and examples ar added for completeness. They will not be part of the exam.
- Let $M = (U_M, V_M)$ be a model structure.
- A variable assignment is a function g:
  VAR $\rightarrow U_M$ that maps variables to elements of U_M.
- g[x/u] stands for the assignment g' which differs from g at most in that g'(x) = u
  - g[x/u](y) = u if x=y
  - g[x/u](y) = g(y) otherwise

Variable Assignment, Examples

	x	y	z	u	…
g	a	b	c	d	…
g[x/a]	a	b	c	d	…
g[y/a]	a	a	c	d	…
g[y/g(z)]	a	c	c	d	…
g[y/a][u/a]	a	a	c	a	…
g[y/a][y/b]	a	b	c	d	…

Interpretation of Terms

- Let $M = (U_M, V_M)$ be a model structure for some language L of predicate logic.
- The function $[[\ \ ]]^{M,g}$ interprets the terms of L as follows:
  - $[[x]]^{M,g} = g(x)$, if x is a variable
  - $[[c]]^{M,g} = V_M(c)$, if c is a constant

Interpretation of Formulae

- $[[R(t_1, ..., t_n)]]^{M,g} = 1$ iff $([[t_1]]^{M,g}, ..., [[t_n]]^{M,g}) \in V_M(R)$
- $[[s = t]]^{M,g} = 1$ iff $[[s]]^{M,g} = [[t]]^{M,g}$
- $[[\neg\varphi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 0$
- $[[\varphi \wedge \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 1$ and $[[\psi]]^{M,g} = 1$
- $[[\varphi \vee \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 1$ or $[[\psi]]^{M,g} = 1$
- $[[\varphi \rightarrow \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = 0$ or $[[\psi]]^{M,g} = 1$
- $[[\varphi \leftrightarrow \psi]]^{M,g} = 1$ iff $[[\varphi]]^{M,g} = [[\psi]]^{M,g}$
- $[[\exists x\Phi]]^{M,g} = 1$ iff there is an $a \in U_M$ s.t. $[[\Phi]]^{M,g[x/a]} = 1$
- $[[\forall x\Phi]]^{M,g} = 1$ iff for all $a \in U_M$, $[[\Phi]]^{M,g[x/a]} = 1$

Example, Revisited

- $[[\exists x(\text{town}(x) \wedge \text{larger\_than}(x, \text{völklingen}))]]^{M,g} = 1$

  there is an $a \in U_M$ s.t. $[[\text{town}(x) \wedge \text{larger\_than}(x, \text{völklingen}))]]^{M,g[x/a]} = 1$

  $\Leftrightarrow [[\text{town}(x)]]^{M, g[x/a]} = 1 \wedge [[\text{larger\_than}(x, \text{völklingen}))]]^{M, g[x/a]} = 1$

  $\Leftrightarrow [[x]]^{M, g[x/a]} \in V_M(\text{town})$ and

  $([[x]]^{M, g[x/a]}, [[\text{völklingen}]]^{M, g[x/a]}) \in V_M(\text{larger\_than})$

  $\Leftrightarrow g[x/a](x) \in V_M(\text{town})$ and

  $(g[x/a](x), V_M(\text{völklingen})) \in V_M(\text{larger\_than})$

  $\Leftrightarrow a \in V_M(\text{town})$ and

  $(a, V_M(\text{völklingen})) \in V_M(\text{larger\_than})$

Truth, Satisfaction, Entailment

- A formula A is true in model structure M
  iff $[[A]]^{M,g} = 1$ for every variable assignment g.
- A formula A is valid ($\models A$)
  - iff A is true in every model structure.
- A set of formulas Γ entails formula A ($\Gamma \models A$) iff A is true in in every model structure M in which all $A \in \Gamma$ are true.

  If all $A \in \Gamma$ are true in a model structure M, we also say that M satisfies (or: simultaneously satisfies) Γ.

Deduction Calculi

- Computing entailment and other logical concepts through semantic interpretation is inefficient and in many cases infeasible.
- Deduction calculi (or proof theoretic systems) provide a strictly syntactic way of checking entailment, through rewrite of logical formulae.
- Formula A is derivable (deducible) from a set of formulas Γ ($\Gamma \vdash A$) in a given deduction system, iff one can obtain A starting from Γ, by using deduction rules and possibly axioms of that deduction system.

A simple Deduction Example

(1) $\forall d\,(dolphin(d) \rightarrow mammal(d) \wedge \neg fish(d))$	Premiss
(2) $dolphin(flipper) \rightarrow mammal(flipper) \wedge \neg fish(flipper)$	Universal Instantiation: $\forall xA \vdash A\,[x/a]$
(3) $dolphin(flipper)$	Premiss
(4) $mammal(flipper) \wedge \neg fish(flipper)$	Modus Ponens: $A, A \rightarrow B \vdash B$ (2), (3)
(5) $mammal(flipper)$	Conjunction reduction (4)

Is Flipper a mammal?

$\forall d\,(dolphin(d) \rightarrow mammal(d) \wedge \neg fish(d))$
$dolphin(flipper) \rightarrow mammal(flipper) \wedge \neg fish(flipper)$ $dolphin(flipper)$
$mammal(flipper) \wedge \neg fish(flipper)$
$mammal(flipper)$

Logical Entailment: ?

$\forall d\,(dolphin(d) \rightarrow mammal(d) \wedge \neg fish(d))$ $\models$ $mammal(flipper)$

Semantic Interpretation

Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf at a time. At birth a bottlenose dolphin calf ...

Flipper is a mammal.

Soundness and Completeness

- So far, deduction systems are just arbitrary rewrite systems for logical formulae.
- Truth-conditional interpretation of the logical formalism enable us to determine whether some given deduction system is
  - sound, i.e., derives only those formula A from a set of premisses Γ which are entailed by Γ.
  - complete, i.e., allows to derive all formulae entailed by Γ.
- In short:
  - Soundness: If $\Gamma \vdash A$, then $\Gamma \models A$.
  - Completeness: If $\Gamma \models A$, then $\Gamma \vdash A$.
- Sound and complete deduction systems derive all and only the truth-conditionally correct entailments.

Theorem Provers

- The problem of FOL entailment checking is very hard: It is even undecidable.
- However, there are automated deduction systems available (called theorem provers, because the original motivation was mathematical theorem proving), which have been optimized through the decades, and have become very efficient.
- So, efficiency is not the problem ...

The Story

- Modelling natural-language inference as deduction in a framework of truth-conditionally interpreted logic appears intuitive and straightforward.
- But: Logical methods are expensive and lack robustness and coverage.