## Computational Linguistics Probabilistic Parsing

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Summer 2016

## Salespeople sold the dog biscuits



| S | $\rightarrow$ NP VP | NP | $\rightarrow$ NP NP |
| ---: | :--- | ---: | :--- |
| VP | $\rightarrow$ V NP | NP | $\rightarrow \mathrm{N}$ |
| VP | $\rightarrow V$ NP NP | DET | $\rightarrow$ the |
| NP | $\rightarrow$ DET $N$ | N | $\rightarrow$ dog |
| NP | $\rightarrow$ DET N N |  | $\ldots$ |

## Ambiguity \& Disambiguation

- Probabilistic disambiguation
choose the one that is most derivation tree if the input sentence is ambiguous (has > 1 derivation trees)
- We need ...
- a probabilistic model of (contex-free) grammar
- methods to estimate probabilities


## Further Motivation

- Natural language is ambiguous
$\Rightarrow$ disambiguation
- Grammar development
$\Rightarrow$ automatically induce grammars
- Efficient search
$\Rightarrow$ compute the most likely parse tree first
■ Robustness


## Probabilistic Context-Free Grammars (PCFG)

- Probabilistic context-free grammar (PCFG)
- a context-free grammar $\langle\mathrm{V}, \Sigma, \mathrm{R}, \mathrm{S}\rangle$
- a funktion $P$ assigning a value $p \in[0,1]$ to each rule
- such that $\sum_{\beta \in \mathrm{v}^{*}} \mathrm{P}(\mathrm{A} \rightarrow \beta)=1$
- $P(A \rightarrow \beta)=$ the conditional probability that symbol $A$ is expanded to $\beta$
- Alternative notations: $P(\beta \mid A), P(A \rightarrow \beta \mid A), A \rightarrow \beta[p]$


## Derivation Trees (Recap)

- Derivarion trees:
- The root node is labeled with the start symbol S
- Leaf nodes are labeled with terminal symbols or $\varepsilon$
- An inner node and their child nodes correspond to the rules that have been used in the derivation
- Parsing:

Compute all derivation trees for a given input

- Probabilistic parsing:

Compute the most likely derivation tree

## Probabilistic Context-Free Grammar (PCFG)

- A PCFG assigns a probability to each derivation tree of a sentence.
- The probability of a derivation tree T is defined as the product of the probabilities of all the rules that have been used to expand the nodes in T :
- $P(T, w)=P(T)=\Pi_{n \in T} P(R(n))$
- $R(n)$ is the rule that has been used to expand node $n$
- Note: $P(T, w)=P(T) P(w \mid T)=P(T)$, because $P(w \mid T)=1$
- The probability of a sentence w is the sum of the probabilities of all its derivation trees:
- $P(w)=\Sigma_{T} P(w, T)$, for $w \in L(G)$


## Salespeople sold the dog biscuits

| $S \rightarrow N P V P$ | [1.0] |
| :---: | :---: |
| $\mathrm{VP} \rightarrow \mathrm{V}$ NP | [0.8] |
| $\mathrm{VP} \rightarrow \mathrm{V}$ NP NP | [0.2] |
| $N P \rightarrow$ DET N | [0.5] |
| $N P \rightarrow N$ | [0.3] |
| $N P \rightarrow$ DET N N | [0.15] |
| $N P \rightarrow N P N P$ | [0.05] |
| DET $\rightarrow$ the | [1.0] |
| $\mathrm{N} \rightarrow$ Salespeople | [0.55] |
| $\mathrm{N} \rightarrow$ dog | [0.25] |
| $\mathrm{N} \rightarrow$ biscuits | [0.2] |
| $V \rightarrow$ sold | [1.0] |



## Salespeople sold the dog biscuits

| $S \rightarrow N P V P$ | [1.0] |
| :---: | :---: |
| $\mathrm{VP} \rightarrow \mathrm{V}$ NP | [0.8] |
| $V P \rightarrow V N P N P$ | [0.2] |
| $N P \rightarrow$ DET N | [0.5] |
| $N P \rightarrow N$ | [0.3] |
| $N P \rightarrow$ DET N N | [0.15] |
| $N P \rightarrow N P N P$ | [0.05] |
| DET $\rightarrow$ the | [1.0] |
| $\mathrm{N} \rightarrow$ Salespeople | [0.55] |
| $\mathrm{N} \rightarrow$ dog | [0.25] |
| $\mathrm{N} \rightarrow$ biscuits | [0.2] |
| $V \rightarrow$ sold | [1.0] |



## Salespeople sold the dog biscuits

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| $\mathrm{N} \rightarrow$ biscuits | [0.2] |
| $V \rightarrow$ sold | [1.0] |



## Probabilistic Context-Free Grammar (PCFG)

- The probability of a sentence $w$ is the sum of the probabilities of all its derivation trees:
- $P(w)=\Sigma_{T} P(w, T)$, for $w \in L(G)$
- A PCFG G is consistent if $\Sigma_{w \in L(G)} P(w)=1$
- Recursion can lead to inconsistent grammars:
- S $\rightarrow$ S S [0.6]
- $\mathrm{S} \rightarrow \mathrm{a}$ [0.4]


## An inconsistent PCFG

- S $\rightarrow$ S S [0.6]/[0.4]

■ $\mathrm{S} \rightarrow \mathrm{a} \quad[0.4] /[0.6]$

- $P\left(a^{i}\right)=\# t r e e s\left(a^{i}\right) \times 0.6^{i-1} \times 0.4^{i}=0.4$
- $P(a)=0.4, P(a a)=0.096, P(a a a)=0.0461, \ldots$
- $P\left(a^{i}\right)=\# t r e e s\left(a^{i}\right) \times 0.4^{i-1} \times 0.6^{i}=0.4$
- $P(a)=0.6, P(a a)=0.144, P(a a a)=0.06912, \ldots$
- Number of trees (\#trees) for $\mathrm{a}^{\mathrm{i}+1}=i$-th Catalan number


## An inconsistent PCFG



## Probabilistic Parsing

■ Language modelling ("inside probabilities") compute the probability that $S \Rightarrow *$ w for an input sentence w:

- $P(w)=\sum_{T} P(w, T)$

■ Probabilistic parsing ("viterbi scores") compute the most likely derivation tree $\mathrm{T}(\mathrm{w})$ for an input sentence w:

- $\mathrm{T}(\mathrm{w})=\arg \max _{\mathrm{T}} \mathrm{P}(\mathrm{T} \mid \mathrm{w})$

$$
\begin{aligned}
& =\arg \max _{T} \frac{P(T, w)}{P(w)} \\
& =\arg \max _{T} P(T)
\end{aligned}
$$

## Properties of PCFGs

- The probability of a (sub) tree is indipendant of
- the context in which the tree occurs
- the node(s) that dominates the tree



## Probabilistic CYK Parsing

- Extend the CYK algorithm:
- $\mathrm{T}[\mathrm{i}, \mathrm{j}, \mathrm{A}]=$ the probability that $\mathrm{A} \Rightarrow * \mathrm{w}_{\mathrm{i}+1} \ldots \mathrm{w}_{\mathrm{j}}$
- Inside probabilities:
- $T[i, j, A]=$ sum of the probabilities of all derivation trees of the substring $\mathrm{w}_{\mathrm{i}+1} \ldots \mathrm{w}_{\mathrm{j}}$
- Probability of a derivation tree (parsing)
- $T[i, j, A]=$ the probability of the most likely derivation
- $\mathrm{B}[\mathrm{i}, \mathrm{j}, \mathrm{A}]=$ the corresponding derivation tree


## CYK (without probabilities)

function CYK (G, $\left.\mathrm{w}_{1} \ldots \mathrm{w}_{\mathrm{n}}\right)$ :

```
for i in 1 ... n do
    T[i-1, i] = { A | A -> wi\in | }
        for j in i - 2 ... 0 do
        T[j, i] = \varnothing
        for k in j + 1 ... i - 1 do
            T[j, i] = T[j, i] u
            {A | A -> B C, B \in T[j,k], C \in T[k, i] }
        done
        done
    done
    if S G T[0, n] then return True else return False
```


## CYK (with probabilities)

function CYK (G, $\mathrm{W}_{1} \ldots \mathrm{~W}_{\mathrm{n}}$ ):
〈initialize T and B )
for i in 1 ... $n$ do
for all nonterminals $A$ in $G$ do $\mathrm{T}[\mathrm{i}-1, \mathrm{i}, \mathrm{A}]=\mathrm{P}\left(\mathrm{A} \rightarrow \mathrm{w}_{\mathrm{i}}\right)$
for j in i - $2 \ldots 0$ do for $k$ in $j+1 \ldots$ i - 1 do
for all $A \rightarrow B C$ do $p r=T[j, k, B] \times T[k, i, C] \times P(A \rightarrow B C)$ if $p r>T[j, i, A]$ then
$\mathrm{T}[\mathrm{j}, \mathrm{i}, \mathrm{A}]=\mathrm{pr}$
$B[j, i, A]=$ (construct subtree〉
return $\langle\mathrm{B}[0, \mathrm{n}, \mathrm{S}]$ and $\mathrm{T}[0, \mathrm{n}, \mathrm{S}]\rangle$

## Learning PCFG Probabilities

- Option \#1
count frequencies of rules in syntactically annotated treebanks (such as the Penn Treebank)
- Option \#2

Inside-outside algorithm (not discussed here)

## Learning PCFG Probabilities

- We are given a syntactically annotated corpus
- annotated corpus = a set of derivation trees
- We can construct a grammar from the treebank by identifying the rules with all "subtrees" of height 1
- Estimating rule probabilities:
- $P(A \rightarrow \alpha)=\frac{\operatorname{count}(A \rightarrow \alpha)}{\sum_{\beta} \operatorname{count}(A \rightarrow \beta)}$
- count $(A \rightarrow \alpha)=$ the number of times the rule $A \rightarrow \alpha$ has been used in all trees in the corpus


## Learning PCFG Probabilities

- A very small treebank:
- $\mathrm{S}_{1}$ : [s [np grass] [vp grows]]
- $\mathrm{S}_{2}$ : [s [np grass] [vp grows] [ap fast]]
- $\mathrm{S}_{3}:$ [s [np grass] [vp grows] [ap slowly]]
- $\mathrm{S}_{4}$ : [s [np bananas] [vp grow]]

■ Rules \& rule probabilities:

- $\mathrm{S} \rightarrow \mathrm{NP}$ VP 2/4
- $\mathrm{S} \rightarrow \mathrm{NP}$ VP AP $2 / 4$
- NP $\rightarrow$ grass 3/4
- ...


## Learning PCFG Probabilities

| Rule | $P(A \rightarrow \alpha)$ |  |
| :--- | :--- | :--- |
| $r_{1}$ | $S \rightarrow$ NP VP | $2 / 4$ |
| $r_{2}$ | $S \rightarrow$ NP VP AP | $2 / 4$ |
| $r_{3}$ | $N P \rightarrow$ grass | $3 / 4$ |
| $r_{4}$ | $N P \rightarrow$ bananas | $1 / 4$ |
| $r_{5}$ | $V P \rightarrow$ grows | $3 / 4$ |
| $r_{6}$ | $V P \rightarrow$ grow | $1 / 4$ |
| $r_{7}$ | $A P \rightarrow$ fast | $1 / 2$ |
| $r_{8}$ | $A P \rightarrow$ slowly | $1 / 2$ |

## Learning PCFG Probabilities

- Probabilities of the sentences:
- $P\left(S_{1}\right)=P\left(r_{1}\right) \times P\left(r_{3}\right) \times P\left(r_{5}\right)=2 / 4 \times 3 / 4 \times 3 / 4=0.28125$
- $\mathrm{P}\left(\mathrm{S}_{2}\right)=\mathrm{P}\left(\mathrm{r}_{2}\right) \times \mathrm{P}\left(\mathrm{r}_{3}\right) \times \mathrm{P}\left(\mathrm{r}_{5}\right) \times \mathrm{P}\left(\mathrm{r}_{7}\right)=0.140625$
- $P\left(S_{3}\right)=P\left(r_{2}\right) \times P\left(r_{3}\right) \times P\left(r_{5}\right) \times P\left(r_{7}\right)=0.140625$
- $P\left(S_{4}\right)=P\left(r_{1}\right) \times P\left(r_{4}\right) \times P\left(r_{6}\right)=0.03125$


## Evaluation

- Coverage: How many sentences are well-formed according to the grammar?
- Accuracy: How many sentences are correctly parsed?
- measured as "relative correctness" wrt. to category label, start and end position (yield) of all constituents (subtrees)
- Labelled precision: percentage of correct subtrees in the parser output
- Labelled recall: percentage of correct subtrees in the gold standard (test corpus)


## Evaluation

- Labelled Precision = C / M
- Labelled Recall $=$ C / N
- where
- C = number of correct constituents produced by the parser
- $M=$ total number of constituents produced by the parser
- $\mathrm{N}=$ total number of constituents in reference corpus


## Binarization

- Replace rules of the form $A \rightarrow A_{1} A_{2} A_{3} \ldots A_{k}[p]$ by
- $A \rightarrow\left\langle A_{1}, \ldots, A_{k-1}\right\rangle A_{k} \quad[p]$
- $\left\langle A_{1}, \ldots, A_{k-1}\right\rangle \rightarrow A_{1} \ldots A_{k-1} \quad[1.0]$
- ... or binarize trees in the treebank before "reading off" the grammar from the trees.


## Problems

- The probability of a (sub) tree is indipendant of
- the context in which the tree occurs
- the node(s) that dominates the tree
- Problems: we want to capture ...

- Lexical dependencies
- Structural dependencies


## Lexical Dependencies

- The two trees differ only in one rule:
- VP $\rightarrow$ VP PP
- NP $\rightarrow$ NP PP

workers



## Lexical Dependencies

- The two trees differ only in one rule:
- VP $\rightarrow$ VP PP
- NP $\rightarrow$ NP PP
- $\Rightarrow$ the grammar will either
- always prefer the 1st rule (VP attachment) or
- always prefer the 2nd rule (NP-attachment)

■ But ...

- Workers dumped sacks into a bin
- Fishermen caught tons of herring
- $\Rightarrow$ Lexikalized PCFG


## Lexical Dependencies

|  | come | take | think | want |
| :--- | ---: | ---: | ---: | ---: |
| $\mathrm{VP} \rightarrow \mathrm{V}$ | $9,5 \%$ | $2,6 \%$ | $4,6 \%$ | $5,7 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V} \mathrm{NP}$ | $1,1 \%$ | $32,1 \%$ | $0,2 \%$ | $13,9 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V} \mathrm{PP}$ | $35,5 \%$ | $3,1 \%$ | $7,1 \%$ | $0,3 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ SBAR | $6,6 \%$ | $0,3 \%$ | $73,0 \%$ | $0,2 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V} \mathrm{S}$ | $2,2 \%$ | $1,3 \%$ | $4,8 \%$ | $70,8 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ NP S | $0,1 \%$ | $5,7 \%$ | $0,0 \%$ | $0,3 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ PRT NP | $0,3 \%$ | $5,8 \%$ | $0,0 \%$ | $0,0 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ PRT PP | $6,1 \%$ | $1,5 \%$ | $0,2 \%$ | $0,0 \%$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |

## Structural dependencies

■ Structural independencies:

- The (probability of an) application of a rule is independent of all other rules in the derivation tree
- NP $\rightarrow$ Pronoun vs. NP $\rightarrow$ Det Noun same probabilities for all occurrences of NP
- But ... (Francis \&al, 1999)
- Subject-NP: 91\% pronouns, 9\% non-pronouns
- Object-NP: 34\% pronouns, 66\% non-pronouns
- (Switchboard corpus, spoken language)
- $\Rightarrow$ Parent annotation


## Structural dependencies

- Some dependencies can be "built into" the category symbols.



## Structural dependencies

- Parent Annotation: nodes are annotated with the label of their parent nodes
- Similar effect compared to conditional probabilities
- P(NP^S $\rightarrow$ PRP)
- P(NP $\rightarrow$ PRP $\mid S$ )
- Compare:
- P(NP-SBJ $\rightarrow$ PRP) - no correspondence to conditional probabilities


## Structural dependencies

- Parent annotation can also be useful for preterminal nodes
- Most frequent adverbs with parent ...
- ADVP - also, now
- VP - not, n't
- NP - only, just
- Penn Treebank - no distinction (same POS) between
- subordinating conjunctions (while, as, if),
- complementizers (that, for)
- prepositions (of, in, from)


## Structural dependencies

- Parent annotation can also be useful for preterminal nodes



## Structural dependencies

- Parent annotation - drawbacks
- the grammar gets larger
- fewer training data for each rule
- reduced generalization ("overfitting")


## Lexical dependencies

- The head of a constituent is the "central" word of a phrase
- Noun - NP
- Verb - VP, S
- Adjektive - AP
- Preposition - PP


## Lexical dependencies

- Lexicalized parsing: annotate nodes with their lexical heads



## Lexical dependencies

## Rule

$r_{1} \quad S_{\text {dumpled }} \rightarrow N_{\text {workers }} V P_{\text {dumped }} \quad 1 / 1$
$r_{2}$ NP workers $\rightarrow$ NNS $_{\text {workers }} \quad 1 / 1$
$r_{3} \quad \mathrm{NP}_{\text {sacks }} \rightarrow \mathrm{NNS}_{\text {sacks }} \quad 1 / 2$
$r_{4} \quad N P_{\text {sacks }} \rightarrow N P_{\text {sacks }}$ PP into $\quad 1 / 2$
$r_{5} \quad N P_{b i n} \rightarrow D T_{a} N N_{b i n}$
1/1

## Lexical dependencies

- Problems:
- this leads to much larger grammars
- its hard to estimate the rule probabilities


## Lexicalized parsing

- Complexity (CYK)
- Runtime: O(|rules|n³),
- Wost case: |rules| = |nonterminals| ${ }^{3}$
- Lexicalized grammars
- Worst case: |rules| = |nonterminals| ${ }^{3} \cdot \mid$ terminals $\left.\right|^{2}$
- |terminals| usually much larger than |nonterminals|
- $\Rightarrow \mathrm{O}\left(\mathrm{n}^{5}\right)$ runtime for typical grammars and input sentences


## Literature

- Jurafsky \& Martin (2009) Speech and Language Processing Kapitel 14.

■ Manning \& Schütze (1999). Foundations of Statistical Natural Language Processing. Kapitel 11 \& 12.

- Eugene Charniak (1993). Statistical Language Learning. Kapitel 5.

