# Computational Linguistics <br> Probabilistic Parsing 

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(Charniak, 1997)

## Salespeople sold the dog biscuits



$$
\begin{array}{rlrl}
\mathrm{S} & \rightarrow \text { NP VP } & \mathrm{NP} & \rightarrow \text { NP NP } \\
\mathrm{VP} & \rightarrow \mathrm{~V} \text { NP } & \mathrm{NP} & \rightarrow \mathrm{~N} \\
\mathrm{VP} & \rightarrow \mathrm{~V} \text { NP NP } & \mathrm{DET} & \rightarrow \text { the } \\
\mathrm{NP} & \rightarrow \text { DET } N & \mathrm{~N} & \rightarrow \text { dog } \\
\mathrm{NP} & \rightarrow \text { DET N N } & & \ldots
\end{array}
$$

## Ambiguity \& Disambiguation

- Probabilistic disambiguation choose the one that is most derivation tree if the input sentence is ambiguous (has > 1 derivation trees)
- We need ...
- a probabilistic model of (contex-free) grammar
- methods to estimate probabilities


## Further Motivation

- Natural language is ambiguous
$\Rightarrow$ disambiguation
- Grammar development
$\Rightarrow$ automatically induce grammars
- Efficient search
$\Rightarrow$ compute the most likely parse tree first
- Robustness


## Probabilistic Context-Free Grammars (PCFG)

- Probabilistic context-free grammar (PCFG)
- a context-free grammar (V, $\Sigma, \mathrm{R}, \mathrm{S}$ )
- a funktion $P$ assigning a value $p \in[0,1]$ to each rule
- such that $\sum_{\beta \in V^{*}} P(A \rightarrow \beta)=1$
- $P(A \rightarrow \beta)=$ the conditional probability that symbol $A$ is expanded to $\beta$
- Alternative notations: $P(\beta \mid A), P(A \rightarrow \beta \mid A), A \rightarrow \beta[p]$


## Derivation Trees (Recap)

- Derivarion trees:
- The root node is labeled with the start symbol S
- Leaf nodes are labeled with terminal symbols or $\varepsilon$
- An inner node and their child nodes correspond to the rules that have been used in the derivation
- Parsing:

Compute all derivation trees for a given input

- Probabilistic parsing:

Compute the most likely derivation tree

## Probabilistic Context-Free Grammar (PCFG)

- A PCFG assigns a probability to each derivation tree of a sentence.
- The probability of a derivation tree $T$ is defined as the product of the probabilities of all the rules that have been used to expand the nodes in T :
- $P(T, w)=P(T)=\Pi_{n \in T} P(R(n))$
- $R(n)$ is the rule that has been used to expand node $n$
- Note: $P(T, w)=P(T) P(w \mid T)=P(T)$, because $P(w \mid T)=1$
- The probability of a sentence $w$ is the sum of the probabilities of all its derivation trees:
- $P(w)=\Sigma_{T} P(w, T)$, for $w \in L(G)$
(Charniak, 1997)


## Salespeople sold the dog biscuits

| S | $\rightarrow$ NP VP | $[1.0]$ |
| ---: | :--- | ---: |
| VP | $\rightarrow$ V NP | $[0.8]$ |
| VP | $\rightarrow$ V NP NP | $[0.2]$ |
| NP | $\rightarrow$ DET N | $[0.5]$ |
| NP | $\rightarrow \mathrm{N}$ | $[0.3]$ |
| NP | $\rightarrow$ DET N N | $[0.15]$ |
| NP | $\rightarrow$ NP NP | $[0.05]$ |
| DET | $\rightarrow$ the | $[1.0]$ |
| N | $\rightarrow$ Salespeople | $[0.55]$ |
| N | $\rightarrow$ dog | $[0.25]$ |
| N | $\rightarrow$ biscuits | $[0.2]$ |
| V | $\rightarrow$ sold | $[1.0]$ |



## Salespeople sold the dog biscuits

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| N | $\rightarrow$ biscuits | $[0.2]$ |
| V | $\rightarrow$ sold | $[1.0]$ |



## Probabilistic Context-Free Grammar (PCFG)

- The probability of a sentence $w$ is the sum of the probabilities of all its derivation trees:
- $P(w)=\Sigma_{T} P(w, T)$, for $w \in L(G)$
- A PCFG G is consistent if $\Sigma_{w \in L(G)} P(w)=1$
- Recursion can lead to inconsistent grammars:
- S $\rightarrow$ S S [0.6]
- $S \rightarrow a \quad[0.4]$


## An inconsistent PCFG

- S $\rightarrow$ S S [0.6]/[0.4]
- $S \rightarrow a \quad[0.4] /[0.6]$
- $P\left(a^{i}\right)=\#$ trees $\left(a^{i}\right) \times 0.6^{i-1} \times 0.4^{i}=0.4$
- $P(a)=0.4, P(a a)=0.096, P(a a a)=0.0461, \ldots$
- $P\left(a^{i}\right)=\#$ trees $\left(a^{i}\right) \times 0.4^{i-1} \times 0.6^{i}=0.4$
- $P(a)=0.6, P(a a)=0.144, P(a a a)=0.06912, \ldots$
- Number of trees (\#trees) for $a^{i+1}=i$-th Catalan number


## An inconsistent PCFG



## Probabilistic Parsing

■ Language modelling ("inside probabilities") compute the probability that $S \Rightarrow^{*} w$ for an input sentence w:

- $P(w)=\Sigma_{T} P(w, T)$

■ Probabilistic parsing ("viterbi scores") compute the most likely derivation tree $\mathrm{T}(\mathrm{w})$ for an input sentence w:

$$
\begin{aligned}
\mathrm{T}(\mathrm{w}) & =\arg \max _{T} \mathrm{P}(\mathrm{~T} \mid \mathrm{w}) \\
& =\arg \max _{T} \frac{\mathrm{P}(\mathrm{~T}, \mathrm{w})}{\mathrm{P}(\mathrm{w})} \\
& =\arg \max _{T} \mathrm{P}(\mathrm{~T})
\end{aligned}
$$

## Properties of PCFGs

- The probability of a (sub) tree is indipendant of
- the context in which the tree occurs
- the node(s) that dominates the tree



## Probabilistic CYK Parsing

- Extend the CYK algorithm:
- $\mathrm{T}[\mathrm{i}, \mathrm{j}, \mathrm{A}]=$ the probability that $\mathrm{A} \Rightarrow{ }^{*} \mathrm{w}_{\mathrm{i}+1} \ldots \mathrm{w}_{\mathrm{j}}$
- Inside probabilities:
- $T[i, j, A]=$ sum of the probabilities of all derivation trees of the substring $w_{i+1} \ldots w_{j}$
- Probability of a derivation tree (parsing)
- $\mathrm{T}[\mathrm{i}, \mathrm{j}, \mathrm{A}]=$ the probability of the most likely derivation
- $\mathrm{B}[\mathrm{i}, \mathrm{j}, \mathrm{A}]=$ the corresponding derivation tree


## CYK (without probabilities)

```
function CYK(G, W1 ... Wn):
    for i in 1 ... n do
    T[i-1, i] = { A | A -> wi | R }
    for j in i - 2 ... 0 do
            T[j, i] = \varnothing
            for k in j + 1 ... i - 1 do
            T[j, i] = T[j, i] u
                { A | A -> B C, B \in T[j,k], C \in T[k, i] }
            done
        done
```

    done
    if \(S \in T[0, n]\) then return True else return False
    
## CYK (with probabilities)

```
function CYK(G, w1 ... wn):
    <initialize T and B)
    for i in 1 ... n do
        for all nonterminals A in G do
        T[i-1, i, A] = P(A A wi}
    for j in i - 2 ... 0 do
        for k in j + 1 ... i - 1 do
            for all A -> B C do
                pr = T[j, k, B] x T[k, i, C] }\timesP(A->BC
                if pr > T[j, i, A] then
                T[j, i, A] = pr
                B[j, i, A] = (construct subtree)
```

return $\langle\mathrm{B}[0, \mathrm{n}, \mathrm{S}]$ and $\mathrm{T}[0, \mathrm{n}, \mathrm{S}]$ )

## Learning PCFG Probabilities

- Option \#1 count frequencies of rules in syntactically annotated treebanks (such as the Penn Treebank)
- Option \#2

Inside-outside algorithm (not discussed here)

## Learning PCFG Probabilities

- We are given a syntactically annotated corpus
- annotated corpus $=$ a set of derivation trees
- We can construct a grammar from the treebank by identifying the rules with all "subtrees" of height 1
- Estimating rule probabilities:
- $P(A \rightarrow \alpha)=\frac{\operatorname{count}(A \rightarrow \alpha)}{\sum_{\beta} \operatorname{count}(A \rightarrow \beta)}$
- $\operatorname{count}(A \rightarrow \alpha)=$ the number of times the rule $A \rightarrow \alpha$ has been used in all trees in the corpus


## Learning PCFG Probabilities

- A very small treebank:
- $\mathrm{S}_{1}$ : [s [np grass] [vp grows]]
- $\mathrm{S}_{2}$ : [s [np grass] [vp grows] [ap fast]]
- $\mathrm{S}_{3}$ : [s [np grass] [vp grows] [Ap slowly]]
- $\mathrm{S}_{4}$ : [s [np bananas] [vp grow]]
- Rules \& rule probabilities:
- $S \rightarrow$ NP VP $2 / 4$
- $\mathrm{S} \rightarrow \mathrm{NP}$ VP AP 2/4
- NP $\rightarrow$ grass 3/4
- ...


## Learning PCFG Probabilities

|  | Rule | $\mathrm{P}(\mathrm{A} \rightarrow \boldsymbol{\alpha})$ |
| :--- | :--- | :--- |
| $r_{1}$ | $\mathrm{~S} \rightarrow$ NP VP | $2 / 4$ |
| $r_{2}$ | $\mathrm{~S} \rightarrow$ NP VP AP | $2 / 4$ |
| $r_{3}$ | $\mathrm{NP} \rightarrow$ grass | $3 / 4$ |
| $r_{4}$ | $\mathrm{NP} \rightarrow$ bananas | $1 / 4$ |
| $r_{5}$ | $\mathrm{VP} \rightarrow$ grows | $3 / 4$ |
| $r_{6}$ | $\mathrm{VP} \rightarrow$ grow | $1 / 4$ |
| $r_{7}$ | AP $\rightarrow$ fast | $1 / 2$ |
| $r_{8}$ | AP $\rightarrow$ slowly | $1 / 2$ |

## Learning PCFG Probabilities

## ■ Probabilities of the sentences:

- $P\left(S_{1}\right)=P\left(r_{1}\right) \times P\left(r_{3}\right) \times P\left(r_{5}\right)=2 / 4 \times 3 / 4 \times 3 / 4=0.28125$
- $P\left(S_{2}\right)=P\left(r_{2}\right) \times P\left(r_{3}\right) \times P\left(r_{5}\right) \times P\left(r_{7}\right)=0.140625$
- $P\left(S_{3}\right)=P\left(r_{2}\right) \times P\left(r_{3}\right) \times P\left(r_{5}\right) \times P\left(r_{7}\right)=0.140625$
- $P\left(S_{4}\right)=P\left(r_{1}\right) \times P\left(r_{4}\right) \times P\left(r_{6}\right)=0.03125$


## Evaluation

- Coverage: How many sentences are well-formed according to the grammar?
- Accuracy: How many sentences are correctly parsed?
- measured as "relative correctness" wrt. to category label, start and end position (yield) of all constituents (subtrees)
- Labelled precision: percentage of correct subtrees in the parser output
- Labelled recall: percentage of correct subtrees in the gold standard (test corpus)


## Evaluation

- Labelled Precision = C / M
- Labelled Recall = C / N
- where
- $\mathrm{C}=$ number of correct constituents produced by the parser
- $M=$ total number of constituents produced by the parser
- $\mathrm{N}=$ total number of constituents in reference corpus


## Binarization

- Replace rules of the form $A \rightarrow A_{1} A_{2} A_{3} \ldots A_{k}[p]$ by
- $A \rightarrow\left\langle A_{1}, \ldots, A_{k-1}\right\rangle A_{k}$ [p]
- $\left\langle\mathrm{A}_{1}, \ldots, \mathrm{~A}_{k-1}\right\rangle \rightarrow \mathrm{A}_{1} \ldots \mathrm{~A}_{\mathrm{k}-1} \quad[1.0]$
- ... or binarize trees in the treebank before "reading off" the grammar from the trees.


## Problems

- The probability of a (sub) tree is indipendant of
- the context in which the tree occurs
- the node(s) that dominates the tree
- Problems: we want to capture ...

- Lexical dependencies
- Structural dependencies


## Lexical Dependencies

- The two trees differ only in one rule:
- VP $\rightarrow$ VP PP
- NP $\rightarrow$ NP PP



## Lexical Dependencies

- The two trees differ only in one rule:
- VP $\rightarrow$ VP PP
- NP $\rightarrow$ NP PP

■ $\Rightarrow$ the grammar will either

- always prefer the 1st rule (VP attachment) or
- always prefer the 2 nd rule (NP-attachment)
- But ...
- Workers dumped sacks into a bin
- Fishermen caught tons of herring
- $\Rightarrow$ Lexikalized PCFG


## (Manning \& Schütze)

## Lexical Dependencies

|  | come | take | think | want |
| :--- | ---: | ---: | ---: | ---: |
| $\mathrm{VP} \rightarrow \mathrm{V}$ | $9.5 \%$ | $2.6 \%$ | $4.6 \%$ | $5.7 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ NP | $1.1 \%$ | $32.1 \%$ | $0.2 \%$ | $13.9 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V} \mathrm{PP}$ | $35.5 \%$ | $3.1 \%$ | $7.1 \%$ | $0.3 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ SBAR | $6.6 \%$ | $0.3 \%$ | $73.0 \%$ | $0.2 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ S | $2.2 \%$ | $1.3 \%$ | $4.8 \%$ | $70.8 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ NP S | $0.1 \%$ | $5.7 \%$ | $0.0 \%$ | $0.3 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ PRT NP | $0.3 \%$ | $5.8 \%$ | $0.0 \%$ | $0.0 \%$ |
| $\mathrm{VP} \rightarrow \mathrm{V}$ PRT PP | $6.1 \%$ | $1.5 \%$ | $0.2 \%$ | $0.0 \%$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |

## Structural dependencies

- Structural independencies:
- The (probability of an) application of a rule is independent of all other rules in the derivation tree
- NP $\rightarrow$ Pronoun vs. NP $\rightarrow$ Det Noun same probabilities for all occurrences of NP
- But ... (Francis \&al, 1999)
- Subject-NP: 91\% pronouns, 9\% non-pronouns
- Object-NP: 34\% pronouns, 66\% non-pronouns
- (Switchboard corpus, spoken language)
- $\Rightarrow$ Parent annotation


## Structural dependencies

- Some dependencies can be "built into" the category symbols.





## Structural dependencies

- Parent Annotation: nodes are annotated with the label of their parent nodes
- Similar effect compared to conditional probabilities
- P(NP^S $\rightarrow$ PRP)
- $\mathrm{P}(\mathrm{NP} \rightarrow \mathrm{PRP} \mid \mathrm{S})$

- Compare:
- P(NP-SBJ $\rightarrow$ PRP) - no correspondence to conditional probabilities


## Structural dependencies

- Parent annotation can also be useful for preterminal nodes
- Most frequent adverbs with parent ...
- ADVP - also, now
- VP - not, n't
- NP - only, just
- Penn Treebank - no distinction (same POS) between
- subordinating conjunctions (while, as, if),
- complementizers (that, for)
- prepositions (of, in, from)


## Structural dependencies

- Parent annotation can also be useful for preterminal nodes




## Structural dependencies

- Parent annotation - drawbacks
- the grammar gets larger
- fewer training data for each rule
- reduced generalization ("overfitting")


## Lexical dependencies

- The head of a constituent is the "central" word of a phrase
- Noun - NP
- Verb - VP, S
- Adjektive - AP
- Preposition - PP


## Lexical dependencies

- Lexicalized parsing: annotate nodes with their lexical heads



## Lexical dependencies

|  | Rule | $P(A \rightarrow \alpha)$ |
| :---: | :---: | :---: |
| $r_{1}$ | $\mathrm{S}_{\text {dumpled }} \rightarrow$ NP ${ }_{\text {workers }} \mathrm{VP}_{\text {dumped }}$ | 1/1 |
| $r_{2}$ | NP workers $\rightarrow$ NNS ${ }_{\text {workers }}$ | 1/1 |
| $\mathrm{r}_{3}$ | NP ${ }_{\text {sacks }} \rightarrow \mathrm{NNS}_{\text {sacks }}$ | 1/2 |
| $\mathrm{r}_{4}$ | $N P_{\text {sacks }} \rightarrow \mathrm{NP}_{\text {sacks }} \mathrm{PP}_{\text {into }}$ | 1/2 |
| $\mathrm{r}_{5}$ | $N P_{\text {bin }} \rightarrow \mathrm{DT}_{\mathrm{a}} \mathrm{NN}_{\text {bin }}$ | 1/1 |
| $\ldots$ | ... | $\cdots$ |

## Lexical dependencies

## ■ Problems:

- this leads to much larger grammars
- its hard to estimate the rule probabilities


## Lexicalized parsing

- Complexity (CYK)
- Runtime: $\mathrm{O}\left(|r u l e s| \mathrm{n}^{3}\right)$,
- Wost case: |rules| = |nonterminals| ${ }^{3}$
- Lexicalized grammars
- Worst case: |rules| = |nonterminals| ${ }^{3}$. |terminals| ${ }^{2}$
- |terminals| usually much larger than |nonterminals|
- $\Rightarrow O\left(n^{5}\right)$ runtime for typical grammars and input sentences


## Literature

- Jurafsky \& Martin (2009) Speech and Language Processing Kapitel 14.
- Manning \& Schütze (1999). Foundations of Statistical Natural Language Processing. Kapitel 11 \& 12.
- Eugene Charniak (1993). Statistical Language Learning. Kapitel 5.

