Computational Linguistics

Latent Spaces and Matrix Factorization

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Goal

Goal:

treat document clustering and word clustering on the same footing (same semantic space)

find low dimensional representations

The word document matrix

Clustering

Document clustering

describe each document by a vector containing the frequencies of the words

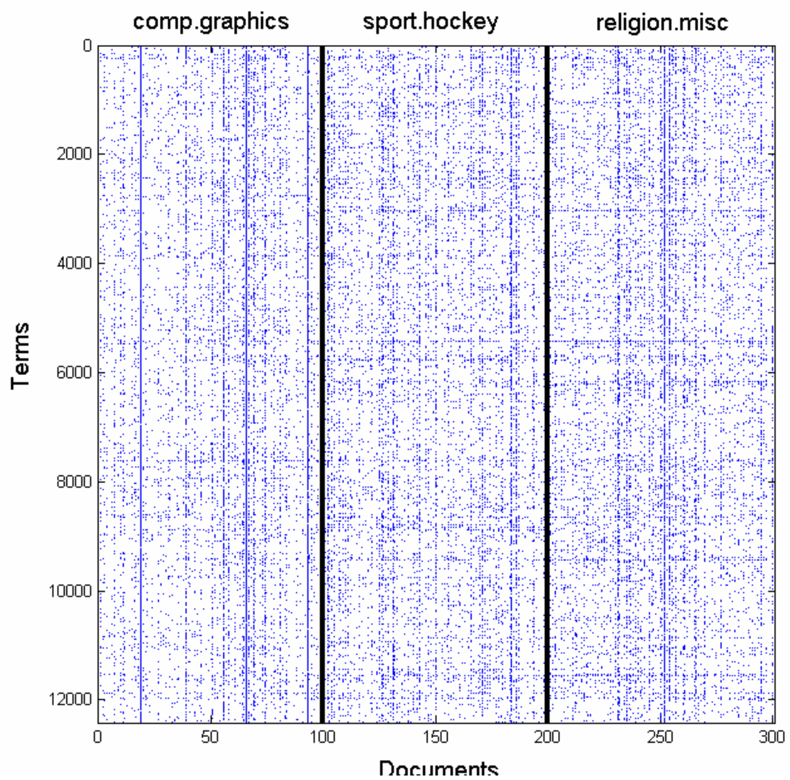
Word clustering

describe each word by a vector containing the frequencies of its occurance in different document

Joint word and document clustering

The word document matrix:

Enter frequency (or tf-idf) for each word and document in a square scheme of numbers (matrix)



Matrices

Matrices

A matrix is an array with two indices

e.g. in a python program this could be `A[i][j]` with i=1..N and j=1...M

When writing, often a subscript notation is used $a_{i,j}$

or a square scheme:

$$A = \begin{pmatrix} a_{1,1} & \dots & a_{1,M} \\ \dots & a_{i,j} & \dots \\ a_{N,1} & \dots & a'_{N,M} \end{pmatrix}$$

Specific example of a 2x3 matrix

$$A = \begin{pmatrix} 2 & -5 & 0.5 \\ -2 & 0.1 & -8 \end{pmatrix}$$

The transpose of a matrix

The two indices are swapped

e.g. in a python program this could be `At[j][i]=A[i][j]` for i=1..N and j=1...M

for the matrices from the previous slide we have:

$$A^t = \begin{pmatrix} a_{1,1} & \dots & a_{1,N} \\ \dots & a_{j,i} & \dots \\ a_{M,1} & \dots & a_{M,N} \end{pmatrix}$$

Specific example of a 2x3 matrix

$$A = \begin{pmatrix} 2 & -5 & 0.5 \\ -2 & 0.1 & -8 \end{pmatrix}$$

What is A^t

Product of two matrices

The elements of a product matrix can be calculated in a python program by

```
for i in range(1,N+1):
    for j in range(1,M+1):
        for k in range(1,K+1):
            C[i][j] = A[i][k]*B[k][j]
```

In math notation $C = A \cdot B$

with $$c_{i,j} = \sum_{k=1}^{K} a_{i,k} b_{k,j}$$

Example see black board

Unit matrix

Unit matrix: the element are the indicator function

$$a_{i,j} = \delta_{i,j}$$

Example:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Often the unit matrix is denoted by a 1

Orthogonal matrices

a matrix A is orthogonal if

$$1 = A^t \cdot A$$

Is the following matrix orthogonal:

$$A = \begin{pmatrix} 0.96 & -0.28 \\ 0.28 & 0.96 \end{pmatrix}$$

more to follow next week