

Computational Linguistics

Lecture 3 – Parsing

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Grammars

- Grammars generate sentences (“words”)

S → NP VP	DET → <i>the</i>	<i>The student works</i>
NP → DET N	DET → <i>a</i>	<i>The student works in the library</i>
NP → NP PP	N → <i>student</i>	<i>The student reads a book</i>
PP → P NP	N → <i>book</i>	<i>The student reads a book in the library</i>
VP → V	N → <i>library</i>	<i>[...]</i>
VP → V NP	V → <i>works</i>	
VP → VP PP	V → <i>reads</i>	
	P → <i>in</i>	

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Context-free Grammars

- Context-free grammar $G = \langle N, T, R, S \rangle$
 - Nonterminal symbols N
 - Terminal symbols T
 - Start symbol $S \in N$
 - Finite set of production rules: $R \subseteq N \times (N \cup T)^*$

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Derivations

- Let $x, y, u, v, w, z \in (N \cup T)^*$
- We write $x \Rightarrow_G y$ iff
 - $x = uvw$
 - $y = uzv$
 - $v \rightarrow z \in R$
- **Derivation of w_n from w_0 :**
 - $w_0 \Rightarrow_G w_1 \Rightarrow_G \dots \Rightarrow_G w_n$
- **Language generated by $G = \langle N, T, R, S \rangle$**
 - $L(G) = \{ w \mid S \Rightarrow_G^* w \}$
 - \Rightarrow_G^* is the reflexive, transitive closure of \Rightarrow_G

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An Example

$S \Rightarrow_G NP VP$
 $\Rightarrow_G DET N VP$
 $\Rightarrow_G the N VP$
 $\Rightarrow_G the student VP$
 $\Rightarrow_G the student V$
 $\Rightarrow_G the student works$

$S \rightarrow NP VP$	$DET \rightarrow the$
$NP \rightarrow DET N$	$DET \rightarrow a$
$NP \rightarrow NP PP$	$N \rightarrow student$
$PP \rightarrow P NP$	$N \rightarrow book$
$VP \rightarrow V$	$N \rightarrow library$
$VP \rightarrow V NP$	$V \rightarrow works$
$VP \rightarrow VP PP$	$V \rightarrow reads$
	$P \rightarrow in$

“the student works” $\in L(G)$

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Another Example

$S \Rightarrow_G NP VP$
 $\Rightarrow_G NP V$
 $\Rightarrow_G NP works$
 $\Rightarrow_G DET N works$
 $\Rightarrow_G the N works$
 $\Rightarrow_G the student works$

$S \rightarrow NP VP$	$DET \rightarrow the$
$NP \rightarrow DET N$	$DET \rightarrow a$
$NP \rightarrow NP PP$	$N \rightarrow student$
$PP \rightarrow P NP$	$N \rightarrow book$
$VP \rightarrow V$	$N \rightarrow library$
$VP \rightarrow V NP$	$V \rightarrow works$
$VP \rightarrow VP PP$	$V \rightarrow reads$
	$P \rightarrow in$

“the student works” $\in L(G)$

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Parse trees

- Context-free grammar $G = \langle N, T, R, S \rangle$
- **Parse trees** are trees where
 - inner nodes are labeled with symbols $\in N$
 - leaf nodes are labeled with symbols $\in T \cup \{\epsilon\}$
 - if v is a node with label A and its child nodes v_1, \dots, v_n are labeled with A_1, \dots, A_n , then $A \rightarrow A_1 \dots A_n$ is a rule of G
 - if v is a leaf node with label ϵ , then v is the only child of its parent node

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Leftmost derivation

- **Leftmost derivation:** replace the leftmost nonterminal symbol in each step of the derivation
- $x \Rightarrow_L y$ iff there are $A \in N$, $a, b \in (N \cup T)^*$, $w \in T^*$ such that
 - $x = wAb$
 - $y = wab$
 - $A \rightarrow a \in R$
- **Rightmost derivation:** analogously

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Theorem (Lewis & Papadimitriou)

- Let $G = \langle N, T, R, S \rangle$ be a context-free grammar
- The following statements are equivalent
 - $A \Rightarrow_G^* w = w_1 \dots w_n$
 - There is a parse tree with root A and yield w
 - There is a leftmost derivation $A \Rightarrow_L^* w$
 - There is a rightmost derivation $A \Rightarrow_R^* w$

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Ambiguity

- $w = w_1 \dots w_n$ may have two or more parse trees.
- The grammar is said to be ambiguous in this case.
- Otherwise, we say that the grammar is unambiguous.

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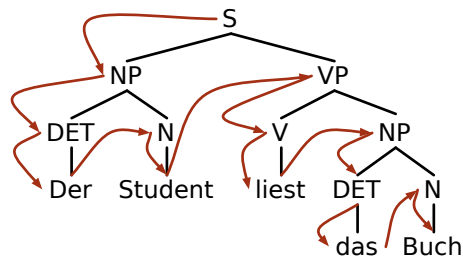
Recognizer & Parser

- **Recognizer**
 - Is $w = w_1 \dots w_n \in L(G)$?
- **Parser**
 - What are the parse trees of $w = w_1 \dots w_n$?

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Basic Parsing Strategies

- **A top-down parser / recognizer ...**
 - starts with the start symbol (= root node)
 - applies production rules “from left to right”
 - and tries to match the input sequence

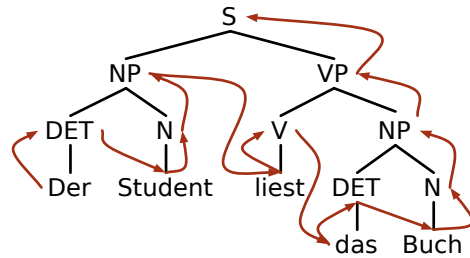


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Basic Parsing Strategies

■ A bottom-up parser / recognizer ...

- starts with the input sequence (= leaf nodes)
- scans the input for subsequences that match the right-hand side of some rule and applies it “from right to left”



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Shift-Reduce Parsing (Bottom-up)

■ Initial configuration for input sequence $w_1 \dots w_n$:

- $\langle [], [w_1, \dots, w_n] \rangle$ — the input that still needs to be processed

stack

■ Accepting configuration

- $\langle [S], [] \rangle$

■ In each step we can perform ...

- shift - move a symbol to the stack
- reduce - apply a matching rule to the topmost elements on the stack

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Shift

- The shift operation moves one symbol to the stack

■ Configuration:

- $\langle [A_1, \dots, A_k], [w_i, w_{i+1}, \dots, w_n] \rangle$

■ New configuration:

- $\langle [A_1, \dots, A_k, w_i], [w_{i+1}, \dots, w_n] \rangle$

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Reduce

- Reduce replaces the topmost symbols on the stack by the lefthand side of a matching rule
- **Configuration:**
 - $\langle [A_1, \dots, A_{j-1}, A_j, \dots, A_k], [w_i, \dots, w_n] \rangle$
- **Rule:**
 - $B \rightarrow A_j, \dots, A_k$
- **New Configuration:**
 - $\langle [A_1, \dots, A_{j-1}, B], [w_i, \dots, w_n] \rangle$

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An Example

$\langle [], [the\ student\ works] \rangle$
 $\Rightarrow_{\text{shift}} \langle [the], [student\ works] \rangle$
 $\Rightarrow_{\text{red}} \langle [DET], [student\ works] \rangle$
 $\Rightarrow_{\text{shift}} \langle [DET\ student], [works] \rangle$
 $\Rightarrow_{\text{red}} \langle [DET\ N], [works] \rangle$
 $\Rightarrow_{\text{red}} \langle [NP], [works] \rangle$
 $\Rightarrow_{\text{shift}} \langle [NP\ works], [] \rangle$
 $\Rightarrow_{\text{red}} \langle [NP\ V], [] \rangle$
 $\Rightarrow_{\text{red}} \langle [NP\ VP], [] \rangle$
 $\Rightarrow_{\text{red}} \langle [S], [] \rangle$

$S \rightarrow NP\ VP$	$DET \rightarrow the$
$NP \rightarrow DET\ N$	$DET \rightarrow a$
$NP \rightarrow NP\ PP$	$N \rightarrow student$
$PP \rightarrow P\ NP$	$N \rightarrow book$
$VP \rightarrow V$	$N \rightarrow library$
$VP \rightarrow V\ NP$	$V \rightarrow works$
$VP \rightarrow VP\ PP$	$V \rightarrow reads$
	$P \rightarrow in$

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Shift or Reduce?

- How can we decide whether we should perform a shift or a reduce operation?
 - For certain (unambiguous) grammars, it is possible to decide this automatically
 - In general \Rightarrow Search

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Python

```
rules = [( 'S', ['NP', 'VP']), ('NP', ['DET', 'N']), ...]

def shift(stack, sent):
    return (stack + [sent[0]], sent[1:])

def reduce(stack, sent, lhs, rhs):
    return (stack[:-len(rhs)] + [lhs], sent)

def matches(stack, rhs):
    for (s, r) in zip(stack[-len(rhs):], rhs):
        if s != r:
            return False
    return True
```

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Python

```
def recognize(sent):
    agenda = [( [], sent)]
    while agenda:
        (stack, sent) = agenda.pop()
        if sent == [] and stack == ['S']:
            return True
        if sntnc != []:
            agenda.append(shift(stack, sent))
        for (lhs, rhs) in rules:
            if len(stack) >= len(rhs):
                if matches(stack, rhs):
                    agenda.append(reduce(stack, sent, lhs, rhs))
    return False
```

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Example – *The student works*

(stack, sent)	agenda
-	⟨[] [the student works]⟩
⟨[] [the student works]⟩	⟨[the] [student works]⟩
⟨[the] [student works]⟩	⟨[DET] [student works] ⟨[the student] [works]⟩
⟨[DET] [student works]⟩	⟨[DET student] [works]⟩ ⟨[the student] [works]⟩
⟨[DET student] [works]⟩	⟨[DET N] [works]⟩ ⟨[DET student works] []⟩ ⟨[the student] [works]⟩
⟨[NP] [works]⟩	⟨[NP] [works]⟩ ⟨[DET N works] []⟩ ⟨[DET student works] []⟩ ...
⟨[NP works] []⟩	⟨[NP works] []⟩ ⟨[DET N works] []⟩ ⟨[DET student works] []⟩ ...
⟨[NP V] []⟩	⟨[NP V] []⟩ ⟨[DET N works] []⟩ ⟨[DET student works] []⟩ ...
⟨[NP VP] []⟩	⟨[NP VP] []⟩ ⟨[DET N works] []⟩ ⟨[DET student works] []⟩ ...
⟨[S] []⟩	⟨[S] []⟩ ⟨[DET N works] []⟩ ⟨[DET student works] []⟩ ...

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Example – *The student reads ...*

- [⇒ Handout]

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Problematic Rules

- Bottom-up parsers cannot deal with certain types of grammars (the parser may not terminate)
- Rules of the form $A \rightarrow \varepsilon$
 - $\langle [A_1, \dots, A_k], [w_i, \dots, w_n] \rangle$
 - $\langle [A_1, \dots, A_k, A], [w_i, \dots, w_n] \rangle$ (reduce)
 - $\langle [A_1, \dots, A_k, A, A], [w_i, \dots, w_n] \rangle$ (reduce)
 - $\langle [A_1, \dots, A_k, A, A, A], [w_i, \dots, w_n] \rangle$ (reduce)
 - [...]

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Problematic Rules

- Bottom-up parsers cannot deal with certain types of grammars (the parser may not terminate)
- Cyclic rules: $A \rightarrow B, B \rightarrow A$
 - $\langle [A_1, \dots, A_k, A], [w_i, \dots, w_n] \rangle$
 - $\langle [A_1, \dots, A_k, B], [w_i, \dots, w_n] \rangle$ (reduce)
 - $\langle [A_1, \dots, A_k, A], [w_i, \dots, w_n] \rangle$ (reduce)
 - $\langle [A_1, \dots, A_k, B], [w_i, \dots, w_n] \rangle$ (reduce)
 - [...]

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Another Problem ...

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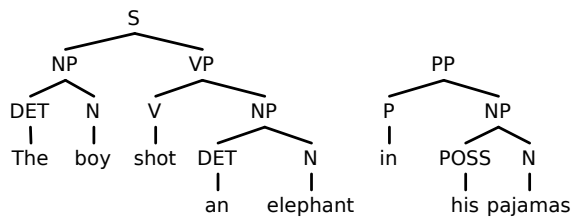
The boy shot an elephant in ...

$\langle [], [the\ boy\ shot\ an\ elephant\ in\ his\ pajamas] \rangle$

$\Rightarrow^* \langle [NP\ VP], [in\ his\ pajamas] \rangle$

$\Rightarrow \langle [S], [in\ his\ pajamas] \rangle$

$\Rightarrow^* \langle [S\ PP], [] \rangle \Rightarrow$ Failure, Backtracking

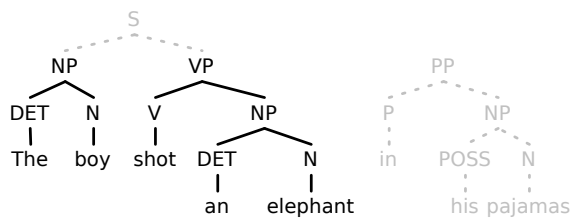


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The boy shot an elephant in ...

$\langle [], [the\ boy\ shot\ an\ elephant\ in\ his\ pajamas] \rangle$

$\Rightarrow^* \langle [NP\ VP], [in\ his\ pajamas] \rangle$



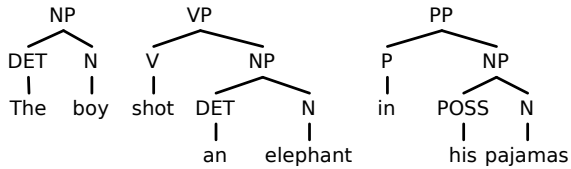
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The boy shot an elephant in ...

<[], [the boy shot an elephant in his pajamas]>

⇒* <[NP VP], [in his pajamas]>

⇒* <[NP VP PP], []>



The boy shot an elephant in ...

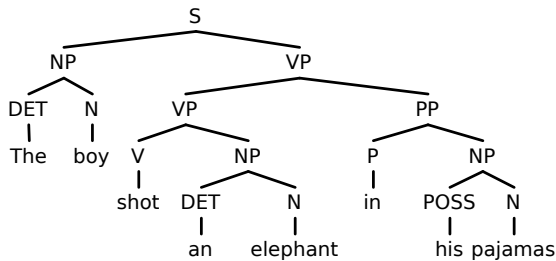
<[], [the boy shot an elephant in his pajamas]>

⇒* <[NP VP], [in his pajamas]>

⇒* <[NP VP PP], []>

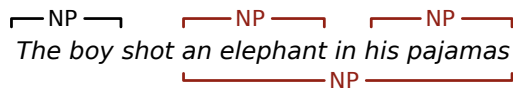
⇒* <[NP VP], []>

⇒* <[S], []>



Dynamic Programming

- **Context-free grammar:** whether or not a rule can be applied does not depend on the context.



- **Chart-Parsing:** store intermediate results for already analysed constituents in a "chart"
- **Charts** are compact representations of all possible analyses ("parse forest")

Chart-Parsing

- **Chart-Parsing:** store intermediate results for already analysed constituents in a “chart”
- **Charts** are compact representations of all possible analyses (“parse forest”)
- Charts can contain
 - complete constituents
 - hypotheses for possible constituents
- Many different chart-parsers:
 - Cocke-Younger-Kasami, Earley, ...

Charts as Matrices

■ $A \in T[i, j]$ iff $A \Rightarrow^* w_{i+1} \dots w_j$

- $S \rightarrow NP VP$ $DET \rightarrow the$
- $NP \rightarrow DET N$ $DET \rightarrow an$
- $NP \rightarrow POSS N$ $N \rightarrow boy$
- $NP \rightarrow NP PP$ $N \rightarrow elephant$
- $PP \rightarrow P NP$ $N \rightarrow pajamas$
- $VP \rightarrow V NP$ $V \rightarrow shot$
- $VP \rightarrow VP PP$ $P \rightarrow in$
- $POSS \rightarrow his$

1	DET								
2	NP	N							
3	∅	∅	V						
4	∅	∅	∅	DET					
5	S	∅	VP	NP	N				
6	∅	∅	∅	∅	∅	P			
7	∅	∅	∅	∅	∅	∅	POSS		
8	S	∅	VP	NP	∅	PP	NP	N	
	0	1	2	3	4	5	6	7	
	The	boy	shot	an	elephant	in	his	pajamas	
	0	1	2	3	4	5	6	7	8

Cocke-Younger-Kasami

- The algorithm by Cocke, Younger, Kasami (CYK) is a simple chart-based bottom-up parser
- **Restriction:** the algorithm can be applied to grammars in Chomsky normal form only:
 - $A \rightarrow w$ (w terminal symbol)
 - $A \rightarrow B C$ (B and C nonterminal symbols)
 - $S \rightarrow \epsilon$ (S start symbol, only if $\epsilon \in L$)
- **Note:** we will assume here that $\epsilon \notin L$, thus the grammar will not contain rules $S \rightarrow \epsilon$

CYK (Recognizer, Pseudo-code)

```
function CYK(G,  $w_1 \dots w_n$ ):  
  for i in 1 ... n do  
    T[i-1, i] = { A | A →  $w_i \in R$  }  
    for j in i - 2 ... 0 do  
      T[j, i] =  $\emptyset$   
      for k in j + 1 ... i - 1 do  
        T[j, i] = T[j, i]  $\cup$   
          { A | A → B C, B ∈ T[j,k], C ∈ T[k, i] }  
      done  
    done  
  done  
  if S ∈ T[0, n] then return True else return False
```

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An Example

- [\Rightarrow blackboard]

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Properties

- **Correct:**
If $S \in T[0, n]$, then $S \Rightarrow^* w_1 \dots w_n$
- **Complete:**
If $S \Rightarrow^* w_1 \dots w_n$, then $S \in T[0, n]$
- **Runtime:**
Polynomial in the input length: $O(n^3)$

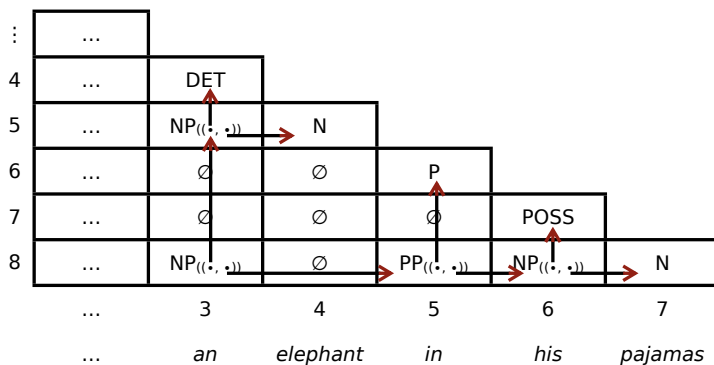
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Recognizer → Parser

- The recognizer can be extended to a parser if we store, for each category A, a list of pointers to other entries in the chart that have been used to derive A

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CYK (Parser)



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Binarization

left binarization(G):

- while G contains rules $A \rightarrow A_1 A_2 A_3 \dots A_k$, $k \geq 3$
- delete the rule from G
- add rule $\langle A_1, \dots, A_{k-1} \rangle \rightarrow A_1 \dots A_{k-1}$
- add rule $A \rightarrow \langle A_1, \dots, A_{k-1} \rangle A_k$

right binarization(G):

- while G contains rules $A \rightarrow A_1 A_2 A_3 \dots A_k$, $k \geq 3$
- delete the rule from G
- add rule $\langle A_2, \dots, A_k \rangle \rightarrow A_2 \dots A_k$
- add rule $A \rightarrow A_1 \langle A_2, \dots, A_k \rangle$

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Implementation variants

- $T[i,j] = T[i,j] \cup \{ A \mid A \rightarrow B C, B \in T[i,k], C \in T[k,j] \}$
 - \Rightarrow can be implemented in different ways
- **Method 1**
 - Iterate over all rules $A \rightarrow B C$
 - Check if $B \in T[i,k]$ and $C \in T[k,j]$
- **Method 2**
 - Iterate over all $B \in T[i,k]$
 - Iterate over all rules $A \rightarrow B C$
 - Check if $C \in T[k, j]$

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Implementierungsvarianten

- $T[i,j] = T[i,j] \cup \{ A \mid A \rightarrow B C, B \in T[i,k], C \in T[k,j] \}$
 - \Rightarrow can be implemented in different ways
- **Method 3**
 - Iterate over all $C \in T[k,j]$
 - Iterate over all rules $A \rightarrow B C$
 - Check if $A \in T[i,k]$
- **Method 4**
 - Iterate over all $B \in T[i,k]$ and $C \in T[k,j]$
 - Check if a rule $A \rightarrow B C$ exists

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Song &al. (EMNLP 2008)

- Experiments mit CYK & Wall Street Journal
- Runtime depends on ...
 - right binarization \Rightarrow method 3 is most efficient
 - left binarization \Rightarrow method 2 is most efficient

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